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Structural Safety 21 (1999) 95–112

STRUCTURAL SAFETY

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A general procedure for first/second-order reliability method (FORM/SORM)

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Abstract

First/second-order reliability method (FORM/SORM) is considered to be one of the most reliable computational methods for structural reliability. Its accuracy is generally dependent on three parameters, i.e. the curvature radius at the design point, the number of random variables and the first-order reliability index. In the present paper, the ranges of the three parameters for which FORM/SORM is accurate enough are investigated. The results can help us to judge when FORM is accurate enough, when SORM is required and when an accurate method such as the inverse fast Fourier transformation (IFFT) method is required. A general procedure for FORM/SORM is proposed which includes three steps: i.e. point fitting limit state surface, computation of the sum of the principal curvatures K_s and failure probability computation according to the range of K_s. The procedure is demonstrated by several examples.

Keywords: Limit state surface; Performance function; FORM/SORM; IFFT; Range of applicability of FORM/SORM; Point-fitting approximation; Reliability index; Hessian matrix; Failure probability

1. Introduction

A fundamental problem in structural reliability theory is the computation of the multi-fold probability integral

$$P_f = \text{Prob}[G(\mathbf{X}) \leqslant 0] = \int_{G(x)\leqslant 0} f(X)\mathrm{d}X \tag{1}$$

where $\mathbf{X} = [X_{1,\dots,}X_n]^T$, in which the superposed T = transpose, is a vector of random variables representing uncertain structural quantities, $f(\mathbf{X})$ denotes the joint probability density function of $\mathbf{X}$, $G(\mathbf{X}$ is the performance function defined such that $G(\mathbf{X})\leqslant 0$, the domain of integration, denotes the failure set, and P_f is the probability of failure. Difficulty in computing this probability

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PII: S0167-4730(99)00008-9

has led to the development of various approximation methods [1], of which the first-order reliability method (FORM) is considered to be one of the most reliable computational methods [2].

FORM is an analytical approximation in which the reliability index is interpreted as the minimum distance from the origin to the limit state surface in standardized normal space (u-space) and the most likely failure point (design point) is searched using mathematical programming methods [3,4]. Because the performance function is approximated by a linear function in u-space at the design point, accuracy problems occur when the performance function is strongly nonlinear [5,6]. The second-order reliability method (SORM) has been established as an attempt to improve the accuracy of FORM. SORM is obtained by approximating the limit state surface in u-space at the design point by a second-order surface [5]. In SORM, the difficult, time consuming portion is the computation of the matrix of second-order derivatives, i.e. the Hessian matrix. To address this problem, an efficient point-fitting algorithm [7,8] is derived, in which the major principal axis of the limit state surface and the corresponding curvature are obtained in the course of obtaining the design point without computing the Hessian matrix; and an alternative point-fitting SORM was developed [16], in which the performance function is directly point-fitted using a general form of the second-order polynomial of standard normal random variables.

If the second-order surface in u-space has been obtained, the failure probability is given as the probability content outside the second-order surface. A numerical integration method was developed by Tvedt [9], and an importance sampling updating method was developed by Hohenbichler et al. [10]. Since the exact computation of the failure probability is quite complicated, numerous studies have contributed to develop some approximations that have closed forms, the accuracy of which is generally dependent on the three parameters of the limit state surface, i.e. the curvature radius R at the design point, the number of random variables n and the first-order reliability index β_F [15]. Breitung [11] has derived an asymptotic formula which approaches the exact failure probability as $\beta_F \rightarrow \infty$ with $\beta_F k_i$, where k_i is a principal curvature at the design point, fixed. Tredt [12] has derived a three-term approximation in which the last two terms can be interpreted as correctors to Breitung's formula. More accurate closed form formulas were derived using McLaurin series expansion and Taylor series expansion by Koyluoglu and Nielsen [13] and Cai and Elishakoff [14]. These formulas generally work well in the case of a large curvature radius and a small number of random variables. However, the rotational transformation and eigenvalue analysis of Hessian matrix for obtaining the principal curvature at the design point, are quite complicated to engineers. To address this, a simple approximation and an empirical second-order reliability index were developed [15], and an IFFT method is proposed as an accurate method to compute the failure probability for the case of extremely small curvature radii or for the case in which the limit state surface can not be approximated by a paraboloid at the design point [16]. Although the ranges of parameters R, n, and β_F for which the simple parabolic approximation and empirical reliability index are accurate, are much larger than those of other SORM formulas, the numerical ranges in detail have not been given. An understanding of these numerical ranges is important because it can help us to judge when the IFFT method may be used.

Another essential problem is the applicable range of FORM. The problem of its accuracy has been examined by many studies through a large number of examples, but the parameter ranges in detail for which it is accurate enough have not been reported according to our knowledge. Without these ranges in detail, it is inconvenient for an engineer to judge whether the results of FORM are accurate enough or not, and when SORM or a more accurate method should be used.

The object of the present paper is to investigate the parameter ranges for which first- and second-order reliability approximations are accurate enough, and propose a general procedure for FORM/SORM.

2. Review of the simple parabolic approximation

The second-order Taylor expansion of a performance function in u-space $G(\mathbf{U})$ at design point $\mathbf{U}^*$ can be expressed as [5,9]:

$$G(\mathbf{U}) = \beta_F - \alpha^T\mathbf{U} + \frac{1}{2}(\mathbf{U} - \mathbf{U}^*)^T\mathbf{B}(\mathbf{U} - \mathbf{U}^*) \tag{2}$$

where

$$\alpha = \frac{\nabla G(\mathbf{U}^*)}{|\nabla G(\mathbf{U}^*)|} \quad \mathbf{B} = \frac{\nabla^2 G(\mathbf{U}^*)}{|\nabla G(\mathbf{U}^*)|} \quad \beta_F = \alpha^T\mathbf{U}^*$$

α is the directional vector at the design point in u-space, $\mathbf{B}$ is the scaled second-order derivatives of $G(\mathbf{U})$ at $\mathbf{U}^*$, known as the scaled Hessian matrix, and β_F is the first-order reliability index.

The sum of the principal curvatures of the limit state surface at the design point can be expressed as [15]:

$$K_S = \sum_{j=1}^{n} b_{jj} - \alpha^T\mathbf{B}\alpha \tag{3}$$

Approximating the limit state surface by a rotational parabolic surface of diameter $2R$, where R is the average principal curvature radius expressed as:

$$R = \frac{n-1}{K_s} \tag{4}$$

The performance functions in u-space, can be expressed simply as:

$$G(\mathbf{U}) = -(u_n - \beta_F) + \frac{1}{2R}\sum_{j=1}^{n-1} u_j^2 \tag{5}$$

The empirical second-order reliability index corresponding to Eq. (5) was obtained as:

$$\beta_S = -\Phi^{-1}\left[\Phi(-\beta_F)\left(1 + \frac{\phi(\beta_F)}{R\Phi(-\beta_F)}\right)^{-\frac{n-1}{2}\left(1+\frac{2K_s}{10(1+2\beta_F)}\right)}\right] \quad K_s \geqslant 0 \tag{6a}$$

$$\beta_s = \left(1 + \frac{2.5K_s}{2n - 5R + 25(23 - 5\beta_F)/R^2}\right)\beta_F + \frac{1}{2}K_s\left(1 + \frac{K_s}{40}\right) \qquad K_s < 0 \tag{6b}$$

where

K_s: The total principal curvature of the limit state surface described in Eq. (3).
R: The average principal curvature radius described in Eq. (4).
n: The number of random variables.
β_F: The first-order reliability index.
β_S: The second-order reliability index.

3. Applicable range for FORM/SORM

3.1. Applicable range for FORM

Considering that the accuracy in engineering application is generally taken as 5%, the following equation is used in the present paper to examine the applicable range for FORM:

$$\left|\Phi(-\beta_S) - \Phi(-\beta_F)\right| \leqslant 0.05\Phi(-\beta_S) \tag{7}$$

where

β_F: The first-order reliability index.
β_S: The second-order reliability index.

Since FORM is only accurate in the case of very large curvature radius and small number of random variables, in the ranges of parameter R, n and β_F for examining the accuracy of FORM, the failure probability is not sensitive to the kind of limit surface (having the same values of R, n and β_F) and the empirical reliability indices Eqs. (6a) and (6b) are accurate enough. Therefore, the Eqs. (6a) and (6b) are used as the second-order reliability index β_S in Eq. (7).

From computations and regressions of Eq. (7), the empirical range of the total principal curvature for which FORM is accurate enough as to be satisfied with Eq. (7) is obtained as

$$|K_s| \leqslant \frac{1}{10\beta_F} \tag{8a}$$

$$\text{or} \quad |R| \geqslant 10\beta_F(n - 1) \tag{8b}$$

where

K_s: The total principal curvature of the limit state surface described in Eq. (3).
R: The average principal curvature radius described in Eq. (4).
n: The number of random variables.

The empirical applicable range of FORM obtained from Eq. (8) with respect to the number of random variables is shown in Fig. 1. along with the computational range directly obtained from Eq. (7), where the first-order reliability index is taken to be 4. From Fig. 1, one can see that the empirical applicable range obtained from Eq. (8) almost coincides with the computational range directly obtained from Eq. (7).

The empirical applicable range of FORM obtained from Eq. (8) with respect to the first-order reliability index is shown in Fig. 2 in the range of β_F is equal to 1–5, along with the computational range directly obtained from Eq. (7), where the number of random variables is taken to be 21. From Fig. 2, one can see that the empirical applicable range obtained from Eq. (8) can be used as approximation of the computational range directly obtained from Eq. (7), especially for a large value of the first-order reliability index. Note, however, that for an extremely small value of β_F, the empirical range obtained from Eq. (8) tends to provide a large error and give a larger range than the computational range. The errors are not as serious as they may at first seem because of the extremely small values of β_F which are out of the range of common use in engineering.

3.2. Applicable range of the simple parabolic approximation

It is very difficult to investigate the applicable range of the simple approximation reviewed in Section 2. However, if a range for which the failure probability is not sensitive to the kind of limit surfaces having the same parameters of curvatures, number of variables and first-order reliability index, the simple approximation of SORM is, of course, accurate in this range. For this purpose, two typical second-order limit state surfaces, i.e. parabolic surface and spherical surface, are investigated in the present paper. The simple approximation of SORM is considered accurate enough when the failure probabilities obtained using the two kinds of limit state surfaces are satisfied with the following equation.

$$\left|P_{fpara} - P_{fsph}\right| \leqslant 0.05 P_{fpara} \tag{9}$$

where

P_{fpara}: The failure probability obtained from parabolic limit state surface.
P_{fsph}: The failure probability obtained from spherical limit state surface.

For investigation on Eq. (9), the performance function of the parabolic limit state surface is taken to be Eq. (5) which is a combination of a standardized normal random variable and a random variable of a central chi-square distribution with n-1 degrees of freedom [5]. The failure probability P_{fpara} is computed using

$$P_{fpara} = \int_0^\infty \Phi\left(\frac{t}{2R} - \beta_F\right) f_{\chi^2_{n-1}}(t)\mathrm{d}t \tag{10}$$

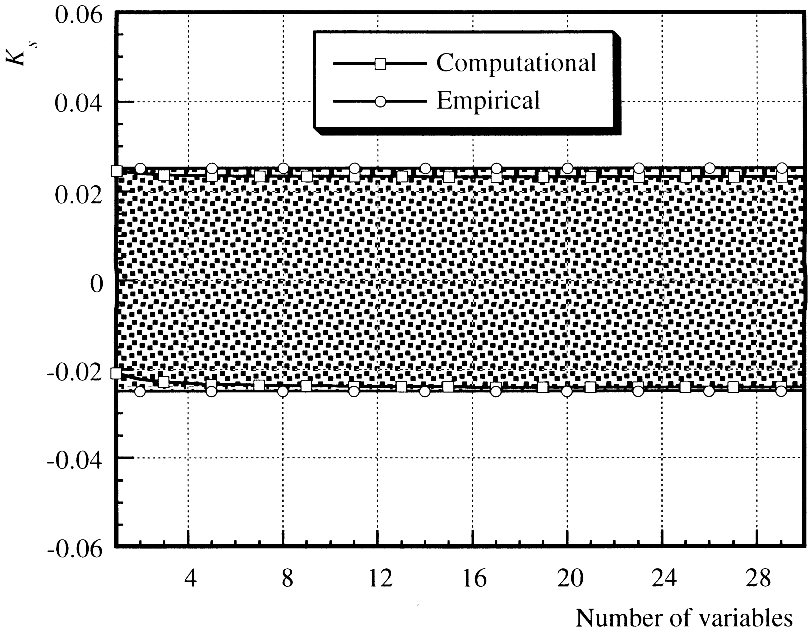

Fig. 1. Comparisons between the computational and empirical applicable range of FORM (with respect to number of variables).

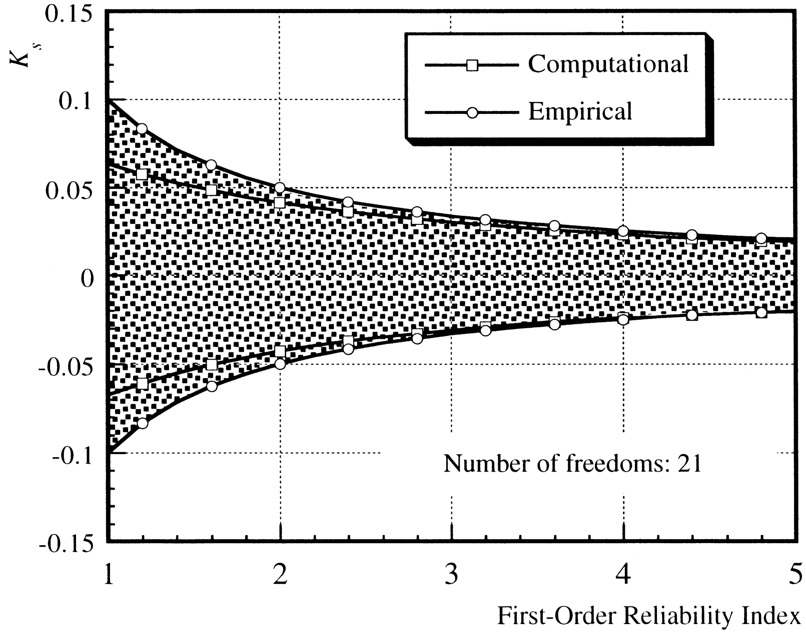

Fig. 2. Comparisons between the computational and empirical applicable range of FORM (with respect to first-order reliability index).

where $f_{\chi^2_{n-1}}(t)$ is the probability density function of central chi-square distribution with n-1 degrees of freedom.

The performance function of the spherical limit state is expressed as:

$$G(\mathbf{U}) = \pm\left[\sum_{j=1}^{n}(u_j - \lambda_j)^2 - R^2\right] \tag{11}$$

The limit state surface of Eq. (11) is a hypersphere with radius R and center at point $(\lambda_j, j = 1, \ldots, n)$. The sign + expresses the limit state surface is convex to the origin and - expresses that concave to the origin. $y = G(\mathbf{U})$ is a random variable having the non-central chi-squared distribution [17] having a non-central parameter of

$$\delta^2 = \sum_{i=1}^{n}\lambda_i^2 = (R \pm \beta_F)^2 \tag{12}$$

and the exact value of the failure probability P_{fsph} is computed directly using this distribution [18,19].

From computations and regressions of Eq. (9), the empirical range of curvatures for which P_{fpara} and P_{fsph} are satisfied with Eq. (9) is obtained as

$$-\frac{1}{10}\left((2 + 0.6\beta_F)\sqrt{n-1} + 3\right) \leqslant K_s \leqslant \frac{2}{5}\left(\sqrt{n-1} + 3\beta_F\right) \tag{13}$$

The computational and empirical applicable range of the simple parabolic approximation with respect to the number of random variables are shown in Fig. 3. where the dashed lines present the results in the case of $\beta_F = 3$ and the solid lines those of $\beta_F = 2$. Fig. 3 shows that the empirical applicable range obtained from Eq. (13) can be used as approximation of the computational range directly obtained from Eq. (9), in all the cases that are combined by $K_S > 0$, $K_S < 0$, $\beta_F = 2$ and $\beta_F = 3$.

3.3. Applicable range of the empirical second-order reliability index

The applicable range of the empirical second-order reliability index is investigated using the following equation:

$$\left|P_{fpara} - \Phi(-\beta_s)\right| \leqslant 0.05 P_{fpara} \tag{14}$$

where

P_{fpara}: The failure probability obtained from parabolic limit state surface.
β_s: The empirical second-order reliability index described in Eq. (6)

The numerical solutions for Eq. (14) for $\beta_F = 2$ are depicted in Fig. 4 and those for $\beta_F = 3$ are depicted in Fig. 5 with comparison with the applicable range of the simple parabolic approximation.

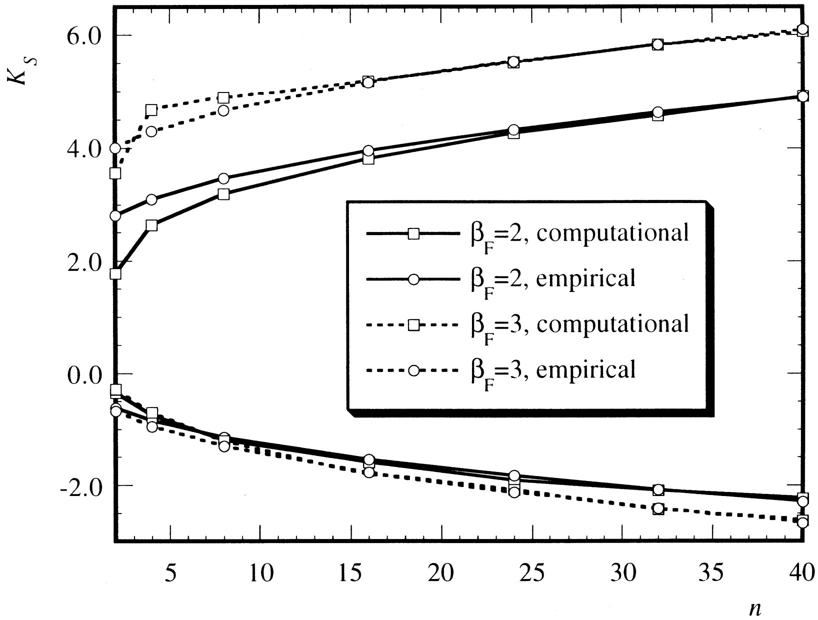

Fig. 3. Applicable range of the simple approximation of SORM.

Figs. 4 and 5 show that the applicable range of the simple parabolic approximation are almost included in the applicable range of the empirical second-order reliability index. That is to say, if a problem can be solved using the simple parabolic approximation, the empirical second-order reliability index is also appropriate for the failure probability computation.

4. General procedure for FORM/SORM

According to the discussion above, the general procedure for FORM/SORM is suggested as three steps: i.e. point fitting limit state surface, computation of the total principal curvature K_S and failure probability computation according to the range of K_S.

4.1. Step 1, point fitting limit state surface

Consider the limit state surface in standard normal space expressed by a performance function $G(\mathbf{U})$. The point-fitted performance function is expressed as a second-order polynomial of standard normal random variables, including $2n+1$ regression coefficients.

$$G'(\mathbf{U}) = a_0 + \sum_{j=1}^{n} \gamma_j u_j + \sum_{j=1}^{n} \lambda_j u_j^2 \tag{15}$$

where a_0, γ_j, and λ_j are $2n+1$ regression coefficients.

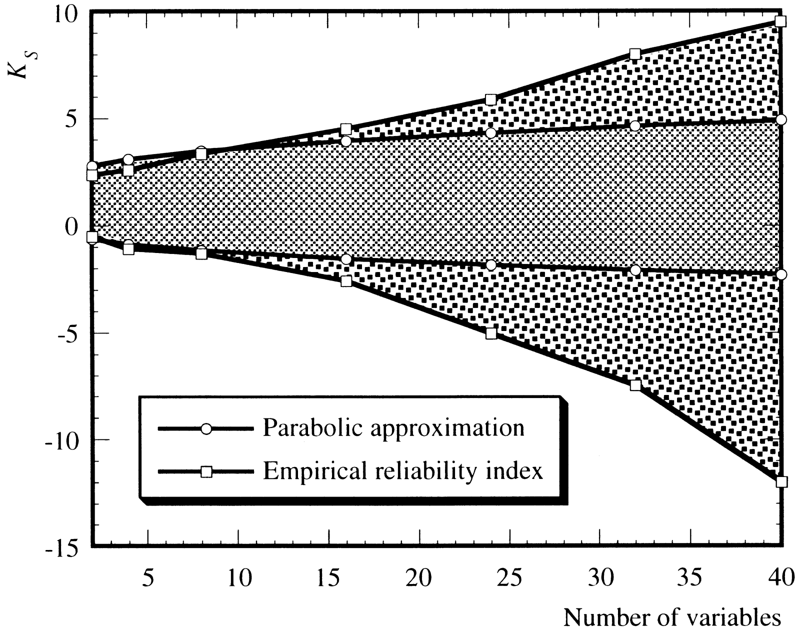

Fig. 4. Applicable range of the empirical reliability index ($\beta_F = 2$).

Using the iterative point-fitting procedure [16] which is essentially an application of the response surface approach [20, 21] in standard normal space, the regression coefficients a_0, γ_j and λ_j can be determined from linear equations of a_0, γ_j and λ_j obtained at each fitting point.

4.2. Step 2, computation of the total principal curvature

After the point-fitted performance function is obtained, the total principal curvature of the limit state surface at the design point $\mathbf{U}^*$ can be computed using Eq. (3). For Eq. (15) K_s is directly expressed as:

$$K_s = \frac{2}{|\nabla G'|}\sum_{j=1}^{n}\lambda_j\left[1 - \frac{1}{|\nabla G'|^2}\left(\gamma_j + 2\lambda_j u_j^*\right)^2\right] \tag{16}$$

where

$$\left|\nabla G'\right| = \sqrt{\sum_{j=1}^{n}\left(\gamma_j + 2\lambda_j u_j^*\right)^2} \tag{17}$$

4.3. Step 3, computation of the failure probability

If the absolute value of K_s is so small that it is satisfied with

$$|K_s| \leqslant \frac{1}{10\beta_F}$$

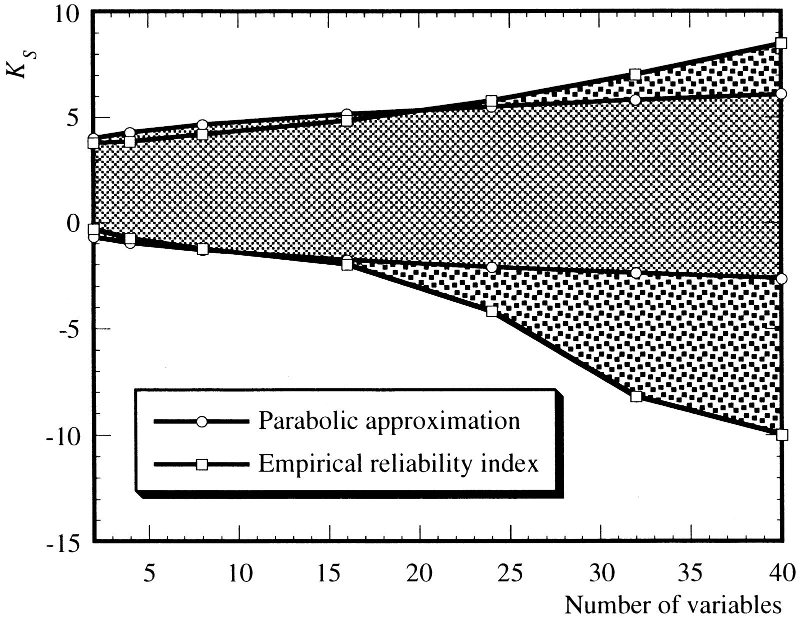

Fig. 5. Applicable range of the empirical reliability index ($\beta_F = 3$).

the failure probability obtained from the FORM corresponding to Eq. (15) is accurate enough.

If the value of K_s satisfies

$$-\frac{1}{10}\left((2+0.6\beta_F)\sqrt{n-1}+3\right)\leqslant K_s\leqslant\frac{2}{5}\left(\sqrt{n-1}+3\beta_F\right)$$

the failure probability should be computed using the empirical second-order reliability index described in Eq. (6).

If the absolute value of K_s is so large that it does not satisfy any case above, the simple parabolic approximation will not be accurate and the failure probability should be computed using more accurate methods such as the IFFT method [16].

The characteristic function corresponding to the point-fitted performance Eq. (15) is expressed as:

$$Q(t)=\exp(ia_0t)\prod_{j=1}^{n}\frac{exp\left(t^2\gamma_j^2/2(1-2it\lambda_j)\right)}{\sqrt{1-2it\lambda_j}} \tag{18}$$

and the failure probability can be readily obtained as:

$$P_f\approx 1-\sum_{r=1}^{N-1}\frac{f(x_r)+f(x_{r+1})}{2}\Delta x \tag{19}$$

where

$$f(x_r) = \frac{t_N - t_1}{2\pi\sqrt{N}} F_r \exp(it_1 x_r) \quad \text{for} \quad x_r \geqslant 0 \tag{20}$$

$$x_r = \frac{2\pi(r-1)}{t_N - t_1}, \qquad \Delta x = 2\pi/(t_N - t_1) \tag{21}$$

$F_r, r = 1, \ldots, N$, are the inverse Fourier coefficients corresponding to the discrete values $Q(t_s), s = 1, \ldots, N$, of the Eq. (18) evenly distributed in the interval of $[t_1, t_N]$, N is the number of discrete data.

In order to visualize the applicable range of FORM/SORM, consider a limit state surface having three random variables and the first-order reliability index of 1.5. From Eqs. (8) and (13), the range of K_s for which FORM is accurate enough is obtained as $-0.067 \leqslant K_s \leqslant 0.067$, and that for which SORM is accurate enough is obtained as $-0.71 \leqslant K_s \leqslant 2.37$. For a limit state surface that have negative curvatures, the corresponding range of the absolute curvature radius is shown in Fig. 6, from which one can see that FORM is accurate enough for $|R| \geqslant 30$, SORM is accurate in the range of $|R| \geqslant 2.8$, when $|R| < 2.8$, the IFFT method is required. For a limit state surface

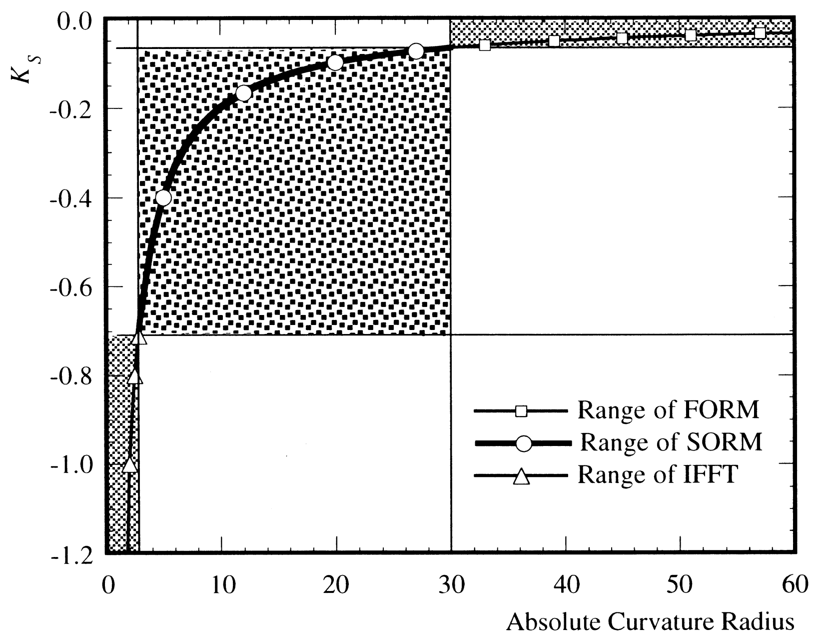

Fig. 6. Applicable range of the FORM/SORM with respect to the curvature radius.

that have positive curvatures, one can similarly obtain that FORM is accurate enough for $R \geqslant 30$, SORM is accurate in the range of $R \geqslant 0.85$, when $R < 0.85$, the IFFT, method is required.

It should be noted that the discussions above are based on the assumption that all the curvatures at the design point have the same sign or are relatively evenly distributed if they have different signs. When the curvatures at the design point have different signs and extremely unevenly distributed, no SORM formulas of closed form can give appropriate results as investigated in Ref. [15]. In this case, the ranges of applicability of FORM/SORM described above can not be used and the IFFT method is generally required.

5. Numerical examples

5.1. A simple example where FORM is accurate enough

The first example considers the following performance function, an elementary reliability model used in many situations,

$$G(\mathbf{X}) = R - S \tag{22}$$

where R is a resistance and S is a load. R is a normal random variable having the coefficient of variance of 0.2, S is a Weibull random variable having a mean value of 100 and coefficient of variance of 0.4. In the following investigations, the exact results are obtained using MCS for 5,000,000 samplings.

Using steps 1 and 2 of the proposed procedure, i.e. point-fitting SORM [16], the total principal curvature of the limit state surface is obtained as Fig. 7, the limit state surface is almost linear and the total principal curvature is inside the range for which FORM is accurate enough. The variation of the reliability index with respect to the central factor of safety obtained using FORM, SORM and MCS are shown in Fig. 8, one can see that the reliability indices obtained by all the methods are almost the same.

5.2. A case where FORM is inadequate but SORM is accurate enough

The second example considers the same performance function in Example 1, but uses different kinds of distribution, i.e. R is a normal random variable having the coefficient of variance of 0.2, S is a lognormal random variable having a mean value of 100 and coefficient of variance of 0.4.

Using steps 1 and 2 of the proposed procedure, the total principal curvature of the limit state surface is obtained as Fig. 9. The limit state surface is much more nonlinear than that in Example 1, so that the total principal curvature is outside the applicable range of FORM but inside that of SORM. The variation of reliability index with respect to central factor of safety is shown in Fig. 10. From Fig. 10, one can see that the FORM has significant errors for this example although the performance function is very simple. The empirical reliability index has a very good agreement with those obtained by MCS.

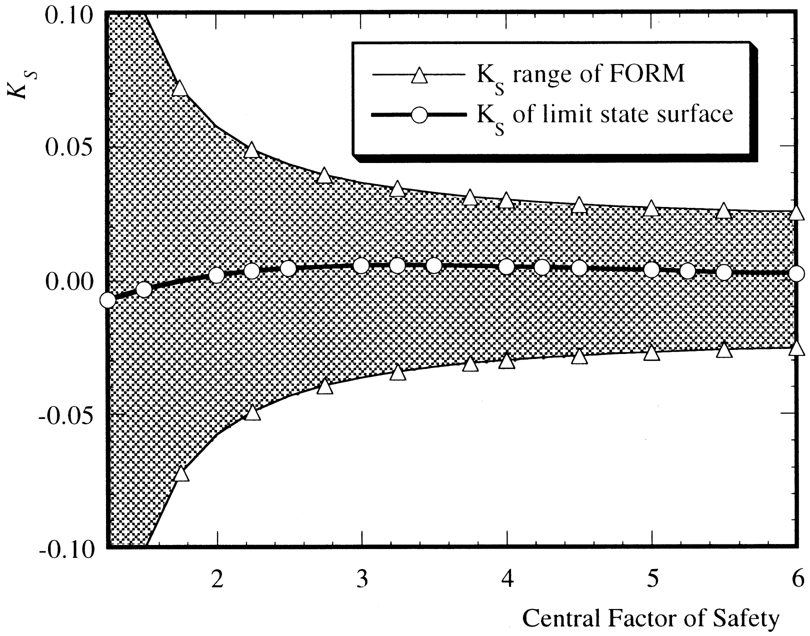

Fig. 7. Sum of the principal curvatures for Example 1.

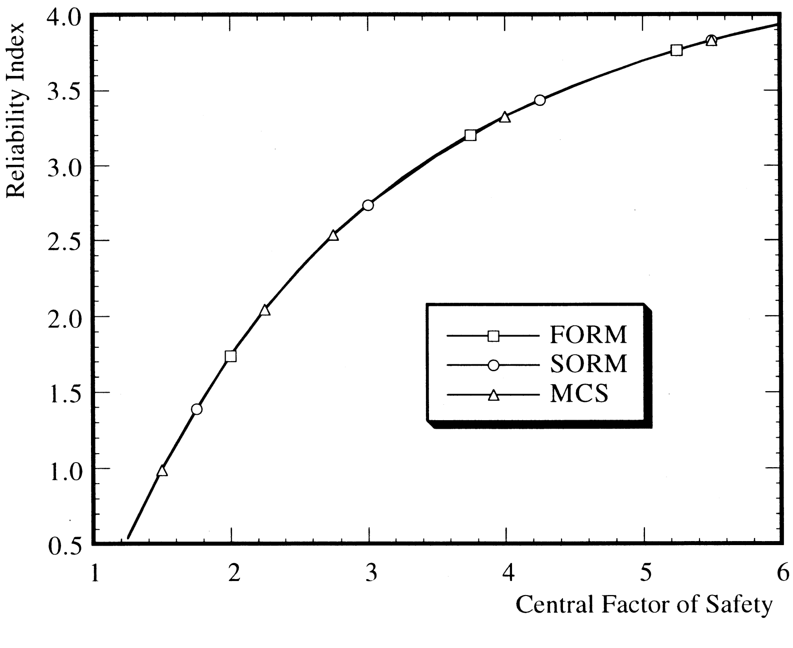

Fig. 8. Variation of reliability index with respect to central factor of safety for Example 1.

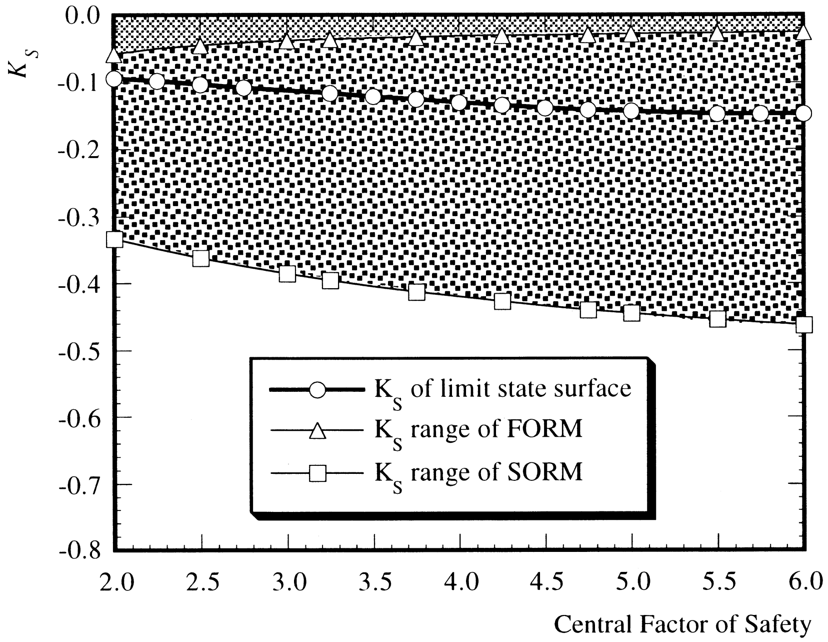

Fig. 9. Sum of the principal curvatures for Example 2.

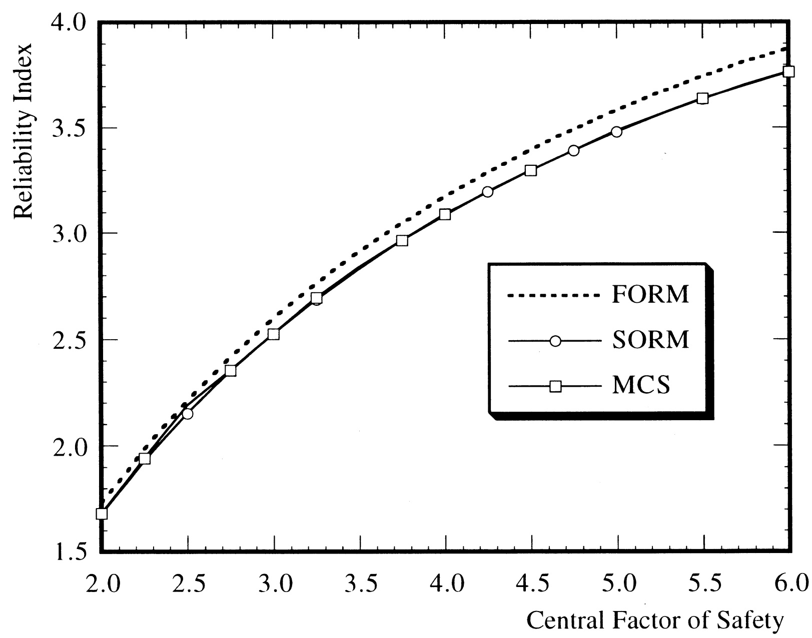

Fig. 10. Variation of reliability index with respect to central factor of safety for Example 2.

5.3. A case where both FORM and SORM are inadequate

The third example considers the following performance function in standardized space, the general case of the practical examples used by Cai [14] and Koyluoglu [13] in their investigations of SORM.

$$G(\mathbf{U}) = r^2 - \sum_{j=1}^{n} \left(u_j - \lambda_j\right)^2 \tag{23}$$

The limit state surface of Eq. (23) is a hypersphere concave to the origin having radius r. $y = G(\mathbf{U})$ is a random variable having the non-central chi-square distribution and the exact values of the failure probability are computed directly using this distribution [19]. In the following investigations, the curvature radius is taken to be 10.0 and the first-order reliability index is taken to be 1.5.

The total principal curvature K_s of the limit state surface is obtained as Fig. 11. K_s is outside the applicable range of FORM even when the number of random variables is equal to 2. K_s is inside the applicable range of SORM when n is under 15 and outside it when n is above 15. The variation of reliability index with respect to the number of variables is shown in Fig. 12. From Fig. 12, one can see that the FORM has significant errors for this example. The empirical reliability index has a good agreement with those obtained by MCS when n is under 15. When n is larger than 15, the empirical reliability index gives significant errors. The IFFT method always gives good results.

5.4. Example 4

Consider the following performance functions which have been used as example 1 by Der Kiureghian [7].

$$G(\mathbf{X}) = x_1, +2x_2 + 2x_3 + x_4 - 5x_5 - 5x_6 \tag{24}$$

The variables x_i are statistically independent and lognormally distributed with the means $\mu_1 = \ldots = \mu_4 = 120, \mu_5 = 50, \mu_6 = 40$, and standard deviations $\sigma_1 = \ldots = \sigma_4 = 12, \sigma_5 = 15$ and $\sigma_6 = 12$.

From the first step of the procedure, the point-fitted performance function is obtained as

$$\begin{aligned} G'(u) = {} & 273.08 + 11.91u_1 + 23.81u_2 + 23.81u_3 + 11.91u_4 - 54.82u_5 - 51.00u_6 \\ & + 0.584u_1^2 + 1.147u_2^2 + 1.147u_3^2 + 0.584u_4^2 - 17.91u_5^2 - 12.08u_6^2 \end{aligned} \tag{25}$$

with corresponding first-order reliability index $\beta_F = 2.3483$ and failure probability $P_F = 0.00943$. Using the second step of the procedure, the total principal curvature is obtained as $K_s = -0.1507$ with corresponding average curvature radius $R = -33.1704$.

Using Eqs. (8) and (13), the K_s range for which FORM is accurate enough is obtained as $|K_s| < 0.0426$, and that for which SORM is accurate enough is obtained as $-1.0623 < K_s < 3.7123$.

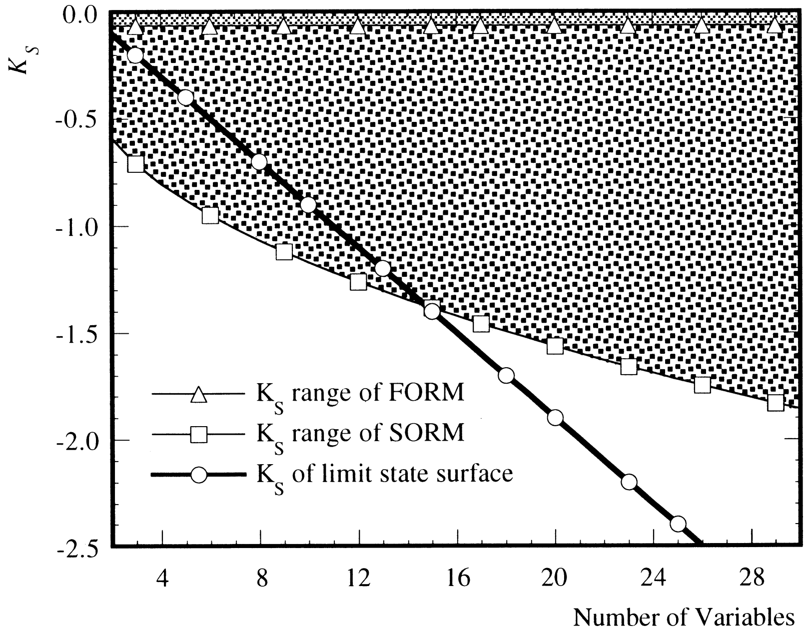

Fig. 11. Sum of the principal curvatures for Example 3.

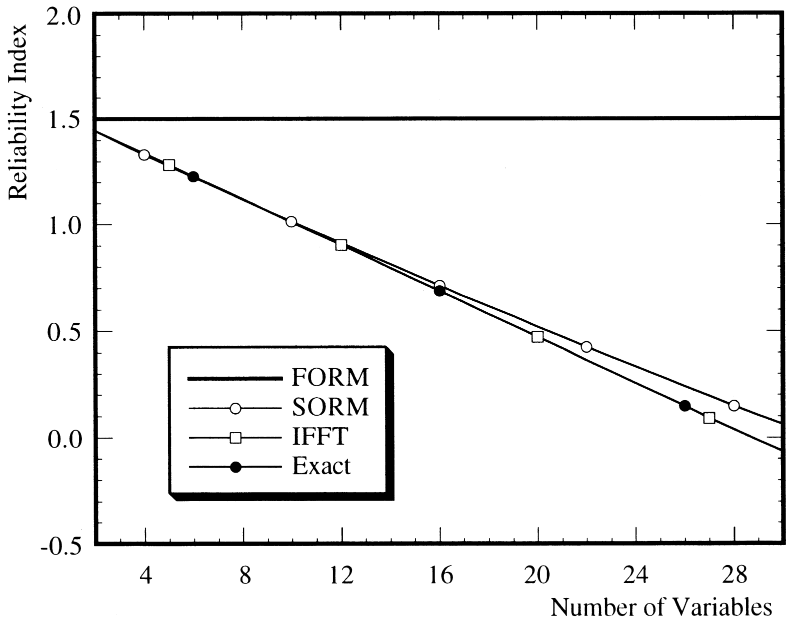

Fig. 12. Variation of reliability index with respect to number of variables for Example 3.

One can see that the total principal curvature of Eq. (25) is inside the K_s range for which SORM is accurate enough. Using Eq. (6), the empirical second-order reliability index is obtained as $\beta_s = 2.2732$ with corresponding failure probability $P_F = 0.01151$ that is a good approximation of the exact result $P_F = 0.0121$ [7].

6. Conclusions

For practical application of FORM/SORM, a general procedure is proposed which includes three steps: i.e. point fitting limit state surface, computation of the total principal curvatures K_s and failure probability computation according to the range of K_s.

The parameter ranges of R, n and β_F for which FORM/SORM is accurate enough are investigated, the results can help us to judge when FORM is accurate enough, when SORM is required and when an accurate method such as IFFT method is required.

It should be noted that the procedure proposed in this paper can be used only for limit state surfaces that have only one design point, a restriction that also applies to other FORM/SORM methods. Otherwise local convergence may occur, and error results may be yielded.

It should be also noted that the ranges of applicability of FORM/SORM can not be used in the case that the curvatures at the design point have different signs and extremely unevenly distributed. For this case, the IFFT method is generally required.

Acknowledgements

The authors wish to thank the referees of this paper for their critical comments and suggestions.

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