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RESPONSE UNCERTAINTY AND TIME-VARIANT RELIABILITY ANALYSIS FOR HYSTERETIC MDF STRUCTURES

Y. G. ZHAO*, T. ONO AND H. IDOTA
Department of Architecture, Nagoya Institute of Technology, Gokiso, Showa-ku, Nagoya 466-8555, Japan

SUMMARY

Response uncertainty evaluation and dynamic reliability analysis corresponding to classical stochastic dynamic analysis are usually restricted to the uncertainties of the excitation. The inclusion of the parameter uncertainties contained in structural properties and excitation characteristics has become an increasingly important problem in many areas of dynamics. In the present paper, a point estimate procedure is proposed for the evaluation of stochastic response uncertainty, and a response surface approach procedure in standard normal space is proposed for analysis of time-variant reliability analysis for hysteretic MDF structures having parameter uncertainties. Using the proposed procedures, the response uncertainties and time-variant reliability can be easily obtained by several repetitions of stochastic response analysis under given parameters without conducting sensitivity analysis, which is considered to be one of the primary difficulties associated with conventional methods. In the time-variant reliability analysis, the failure probability can be readily obtained by improving the accuracy of the first-order reliability method using the empirical second-order reliability index. The random variables are divided into two groups, those with CDF and those without CDF. The latter are included via the high-order moment standardization technique. A numerical example of a 15F hysteretic MDF structure that takes into account uncertainties of four structural parameters and three excitation characteristics is performed, based on which the proposed procedures are investigated and the effects of parameter uncertainties are discussed. Copyright © 1999 John Wiley & Sons, Ltd.

KEY WORDS: parameter uncertainty; response uncertainty; time-variant reliability; hysteretic MDF structure; point-estimates; response surface approach

1. INTRODUCTION

The response uncertainty evaluation and dynamic reliability analysis corresponding to the classical stochastic dynamic analysis are usually restricted to the uncertainties of the excitation. However, it has recently been recognized that the model parameters such as geometry material properties and excitation characteristics, are often poorly understood and the inclusion of these parameter uncertainties has become an increasingly important problem in many areas of

*Correspondence to: Y. G. Zhao, Department of Architecture, Nagoya Institute of Technology, Gokiso, Showa-ku, Nagoya 466-8555, Japan.

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dynamics.[1−5] The present paper attempts to evaluate the response uncertainty and dynamic reliability of hysteretic MDF structures affected by the parameter uncertainties included in the structural properties and excitation characteristics.

For response uncertainty evaluation, early research monographs in this field, addressing both static and dynamic problems, include the structural parameters by adopting series expansions in order to evaluate the structural response.[1,2] In the framework of dynamic stochastic response, studies of uncertain linear systems with deterministic loads may be classified into statistical frequency-domain analysis[6] and time-domain analysis.[7] The conventional approach in the dynamic analysis of structural systems with stochastic uncertainties is based on the series expansion of stochastic quantities with respect to stochastic uncertainties and evaluates the first- and second-order moments of the response by solving deterministic equations once the perturbation method is applied. Some recent studies[8,9] have shown that assuming the first-order approximation of the mean response to be coincident with the deterministic solutions, obtained by fixing the stochastic mean value, can cause significant error when the coefficients of variation of stochastic parameters are relatively large. Therefore, in order to solve static problems, an improved approach that takes into account the first- and second-order probabilistic information of stochastic parameters for computing the mean solution has been proposed. In the improved approach, the variance and covariance of the solutions are calculated using the improved mean solution rather than the mean solution.

Another improved approach[10] has been proposed for evaluating in the time domain the statistical moments of the response of linear systems subject to time-variant deterministic input. This method requires (1) a second-order Taylor series expansion with respect to the uncertain parameters, (2) introduction of the state vector space in order to obtain the first-order differential equations and (3) adoption of the improved statistical moment into the first-order deterministic differential equations.

All of these methods, which are referred to here as series expansion methods, have the following weaknesses:

(1) The sensitivity of the response is required. Obtaining the sensitivity of the response, however, is not always easy because the response analysis includes complicated procedures such as eigenvalue analysis and the solution of line equations.
(2) Application to non-linear systems is difficult. This is partially due to the fact that the sensitivity analysis in a non-linear system is much more difficult than that in a linear system.
(3) Only the case of deterministic input, or the parameter uncertainties contained in structural properties are considered. In reality however, evaluation of response uncertainty due to both the excitation characteristics and the structural properties under random process input is also required, which is complicated if done using series expansion methods.

In order to evaluate the uncertainties of stochastic response due to parameter uncertainties, the present paper proposes a point estimation procedure, in which the stochastic response, including parameter uncertainties, is obtained by several repetitions of stochastic response analysis under given parameters. In the case of non-linear dynamic analysis under stochastic excitation, the proposed procedure can be easily performed, without conducting any sensitivity analysis, which is considered to be one of the primary difficulties associated with conventional methods.

On the other hand, for time-variant reliability analysis, to include parameter uncertainties, one can first solve the problem with given parameters and then integrate over the system parameters to find the overall reliability. However, the computation could become excessive since repeated solutions of stochastic structural response are required, and so an approximate method having good accuracy and numerical efficiency is needed. For a linear oscillator, a First-Order Reliability Method (FORM) was proposed by Hohenbichler and Rackwitz in which the distribution of the maximum peak of the response is used.[11] This method has been used in the case of non-stationary excitation by Balendra and Quek.[12,13] For general problems, reliability estimation methods have been proposed by Gueri and Rackwitz,[14] for which auxiliary variables are introduced so that FORM can be used. A more general discussion of time-variant structural reliability analysis is provided by Wen and Chen,[4] where only one auxiliary random variable is introduced and a performance function was proposed. This method has been applied to system reliability by Wen and Chen.[15,16] Incorporating these methods, a nested FORM has been proposed by Madsen and Tvedt[17] based on the discussion of the calculation of the failure probability with given parameters. In these methods, which enable an evaluation of the dynamic structural reliability that includes parameter uncertainties to be performed using FORM, calculating the gradients of the performance function is an important step. Because it is not always easy to obtain these gradients,[18] a spectral stochastic finite element formulation using polynomial chaoses has been proposed[2,3] and the Response Surface Approach (RSA) has recently been introduced[19] to avoid the difficulties in obtaining the gradients.

Because time-variant reliability in the above methods is conducted using FORM, the problem of accuracy will arise for the case of a strong non-linear performance function. To improve the analytical accuracy, the Second-Order Reliability Method (SORM) may be used, but the computation of Hessian matrix and its the rotational transformation are necessary.[20,21] On the other hand, in all the above methods, the random variables that express the parameter uncertainties are generally expressed as continuous random variables that have a known Cumulative Distribution Function (CDF). In reality however, due to the lack of statistical data, the CDF of some random variables may be unknown, and their probabilistic characteristics may be expressed using only statistical moments.

In order to improve the two weaknesses described above, a computational procedure of RSA in standard normal space is proposed. In this procedure, the time-variant reliability analysis is conducted through several repetitions of dynamic reliability analysis with given parameters, and without any sensitivity analysis. Because the response surface is directly obtained as a polynomial of standard normal random variables, neither a complicated computation of the Hessian matrix is needed, nor is it necessary to carry out its rotational transformation and eigenvalue analysis. In order to include the random variables having no CDF, the random variables included in the analysis model are divided into two groups, those having continuous CDF, such as stiffness, damping, and strength, and those having no CDF, such as excitation characteristics. Random variables having no CDF are included via the High-Order Moment Standardization Technique (HOMST), the use of which requires almost no extra effort.

2. NON-LINEAR STOCHASTIC RESPONSE AND RELIABILITY ANALYSIS WITH DETERMINISTIC PARAMETERS

The uncertainties in classical stochastic dynamic analysis are usually restricted to excitation. For hysteretic MDF structures, the non-linear random vibration analysis is generally conducted using equivalent linearization,[22,23] and the equivalent stiffness and equivalent damping ratio are determined according to random vibration theory using the following equations[24]

$$K_e(\gamma) = \frac{K_0}{\gamma}\left[(1-\alpha)(1+\ln\gamma)+\alpha\gamma\right] \tag{1}$$

$$\xi_n(\gamma) = \xi_0 + 0{\cdot}15\left[1-\frac{1}{\gamma}\sqrt{1+\alpha(\gamma-1)}\right] \tag{2}$$

where K_e is the equivalent stiffness, ξ_n is the equivalent damping ratio for the nth mode, α is the stiffness ratio, γ is the ductility ratio, K_0 is the initial stiffness and ξ_0 the damping ratio.

Assume the ground acceleration is idealized as a segment of finite duration of a stationary Gaussian process having mean of zero. Using the random mode decomposition method combined with the mean-value response spectrum,[25–27] the deviation of the stochastic response is obtained as

$$\sigma_r^2 = \sum_i \sum_j B_i B_j \, \rho_{0,ij} \sqrt{\lambda_{0,ii}\lambda_{0,jj}} \tag{3}$$

where B_i is the participation factor of the ith mode obtained by dynamic analysis of structures, $\rho_{0,ij}$ is the correlation coefficient between the ith and jth modes, which can be obtained from random vibration analysis, and $\lambda_{0,ii}$ is the spectral moment of ith mode, which can be obtained from the mean-value response spectrum.

Using the result of the standard deviation of response σ_r obtained from random vibration analysis, the mean value and standard deviation of the maximum response are obtained using the following equations:

$$\mu_m = \sigma_r\left[\sqrt{2\ln vD} + \frac{0{\cdot}5772}{\sqrt{2\ln vD}}\right] \tag{4}$$

$$\sigma_m = \frac{\pi\sigma_r}{\sqrt{12\ln vD}} \tag{5}$$

where μ_m and σ_m are the mean value and standard deviation, respectively, of the maximum response, v is the mean cross ratio, and D is the duration of ground motion.

After obtaining the deviation of the stochastic response, failure probability is defined as the first passage probability of the maximum response, the probability distribution of which is assumed to be given by

$$F_r(u) = \exp\left\{-vD\exp\left[-\frac{1}{2}\left(\frac{u}{\sigma_r}\right)^2\right]\right\} \tag{6}$$

where v is the cross ratio, and σ_r is the standard deviation of the stochastic response obtained from stochastic vibration analysis.

3. STOCHASTIC RESPONSE INCLUDING PARAMETER UNCERTAINTIES

3.1. Stochastic response with parameter uncertainties

In the analysis described in the previous section, all input parameters, including structural properties and excitation characteristics, are assumed to be deterministic. When these parameter uncertainties are included in the analysis, the parameters are expressed as random variables **X** and the maximum response can be expressed as a function of **X**

$$R_{\max} = R_{\mathrm{m}}(\mathbf{X}) \tag{7}$$

In evaluating the response uncertainty due to the parameter uncertainties, the method generally employed is to expand $R_{\max}$ to a Taylor series and conduct the evaluation using a first-order approximation[1,28] as shown below:

$$\sigma_{\mathrm{M}}^2 = \sum_k \sum_j \frac{\partial R_{\mathrm{m}}(\mathbf{X})}{\partial x_k} \frac{\partial R_{\mathrm{m}}(\mathbf{X})}{\partial x_j} \rho_{kj} \sigma_k \sigma_j \tag{8}$$

where σ_k is the standard deviation of x_k and ρ_{ij} is the correlation coefficient of x_k and x_j.

No explicit description of $R_{\max}$ is given in random vibration theory (only its mean value μ_{m} and standard deviation σ_{m}, expressed by equations (4) and (5), are given). The gradients of $R_{\max}$ are difficult to obtain because the random vibration analysis is non-linear and the gradient analysis procedure includes several complicated procedures, such as the inverse distribution function, eigenvalue analysis, the computation of the participation factor and the spectral moments.[18] In order to avoid the difficulties associated with the computation of gradients, the present paper will evaluate the mean value μ_{M} and standard deviation σ_{M} of $R_{\max}$ utilizing the computational results of equations (4) and (5).

After including the parameter uncertainties described by random variables **X**, the mean value μ_{m} and standard deviation σ_{m}, expressed in equations (4) and (5) become functions of **X**, denoted by $\mu_{\mathrm{m}}(\mathbf{X})$ and $\sigma_{\mathrm{m}}(\mathbf{X})$, respectively. For a group of given values of **X**, the values of the functions $\mu_{\mathrm{m}}(\mathbf{X})$ and $\sigma_{\mathrm{m}}(\mathbf{X})$ can be obtained using the method described in the previous section. To include the uncertain parameters **X**, functions $\mu_{\mathrm{m}}(\mathbf{X})$ and $\sigma_{\mathrm{m}}(\mathbf{X})$ can be computed for given **X**, after which these functions can be integrated over the entire area for which **X** is defined to obtain the mean value μ_{M} and standard deviation σ_{M} of the maximum response.

Since

$$\mu_{\mathrm{m}}(\mathbf{X}) = \int R_{\mathrm{m}}(\mathbf{X}) f(R_{\mathrm{m}} \,|\, \mathbf{X})\, \mathrm{d}R_{\mathrm{m}} \tag{9}$$

$$\sigma_{\mathrm{m}}^2(\mathbf{X}) = \int (R_{\mathrm{m}}(\mathbf{X}) - \mu_{\mathrm{m}}(\mathbf{X}))^2 f(R_{\mathrm{m}} \,|\, \mathbf{X})\, \mathrm{d}R_{\mathrm{m}} \tag{10}$$

μ_{M} and σ_{M} can be expressed as the following equations:

$$\mu_{\mathrm{M}} = \int R_{\mathrm{m}}(\mathbf{X}) f(R_{\mathrm{m}} \,|\, \mathbf{X}) f(\mathbf{X})\, \mathrm{d}R_{\mathrm{m}}\, \mathrm{d}\mathbf{X} = E[\mu_{\mathrm{m}}(\mathbf{X})] \tag{11}$$

$$\sigma_{\mathrm{M}}^2 = \int (R_{\mathrm{m}}(\mathbf{X}) - \mu_{\mathrm{M}})^2 f(R_{\mathrm{m}} \,|\, \mathbf{X}) f(\mathbf{X})\, \mathrm{d}R_{\mathrm{m}}\, \mathrm{d}\mathbf{X} = E[\sigma_{\mathrm{m}}^2(\mathbf{X})] + E[\mu_{\mathrm{m}}^2(\mathbf{X})] - \mu_{\mathrm{M}}^2 \tag{12}$$

where

$$E[\cdot(\mathbf{X})] = \int_x \cdot(\mathbf{X}) f(\mathbf{X})\,\mathrm{d}\mathbf{X} \tag{13}$$

The distribution $f(R_m \mid \mathbf{X})$ is presented only in order to facilitate description, and is not required for the evaluation.

In order to evaluate $E[\mu_m(\mathbf{X})]$, $E[\mu_m^2(\mathbf{X})]$, and $E[\sigma_m^2(\mathbf{X})]$, Monte-Carlo simulation or direct integration may be used. A large number of repetitions of non-linear stochastic analysis is required. Another general approximation is obtained from the Taylor expansion of these functions,[28] in which the previously described difficulties in the computation of derivatives described above will be encountered. The response surface approach can be used to avoid the difficulties in computation of derivatives. However, according to our computational experience, the results of this approximation depend strongly on the fitting points. Therefore, an approximation method that is both efficient and accurate is required. The present paper evaluates $E[\mu_m(\mathbf{X})]$, $E[\mu_m^2(\mathbf{X})]$, and $E[\sigma_m^2(\mathbf{X})]$ using the method of point estimates.

3.2. *Concept of point estimates*

The method of point estimates was proposed by Rosenblueth[29] for estimating the first few moments of a function of random variables. This method uses a weighted sum of the function evaluated at a finite number of points. The weights and points at which the function is evaluated are chosen as the weights and points at which the variable itself must be evaluated to give the correct first few moments of the variable itself, i.e. the estimating points $x_1, x_2, \ldots, x_m$ and the corresponding weights $p_1, p_2, \ldots, p_m$ are selected using the following equations:

$$\sum_{i=1}^{m} p_i = 1 \tag{14}$$

$$\sum_{i=1}^{m} p_i x_i = \mu_x \tag{15}$$

$$\sum_{i=1}^{m} p_i (x_i - \mu_x)^n = \int (x - \mu_x)^n f(x)\,\mathrm{d}x \tag{16}$$

where $f(x)$ is the PDF of x, and μ_x is the mean value of x.

For a three-point estimate, let the concentrations be p_-, and x_-, p_0 and x_0, and p_+ and x_+. Gorman[30] derived the following equations for x_-, x_0, x_+ and p_-, p_0, p_+:

$$p_- = \frac{1}{2}\left(\frac{1 + \alpha_{3x}/\theta}{\alpha_{4x} - \alpha_{3x}^2}\right) \tag{17}$$

$$p_0 = 1 - \frac{1}{\alpha_{4x} - \alpha_{3x}^2} \tag{18}$$

$$p_+ = \frac{1}{2}\left(\frac{1 - \alpha_{3x}/\theta}{\alpha_{4x} - \alpha_{3x}^2}\right) \tag{19}$$

$$x_- = \mu_x - \frac{\sigma_x}{2}(\theta - \alpha_{3x}) \tag{20}$$

$$x_0 = \mu_x \tag{21}$$

$$x_+ = \mu_x + \frac{\sigma_x}{2}(\theta + \alpha_{3x}) \tag{22}$$

where

$$\theta = (4\alpha_{4x} - 3\alpha_{3x}^2)^{1/2} \tag{23}$$

For a function of $y = y(x)$, the kth central moment of y can be calculated using x_-, x_0, x_+, p_-, p_0, and p_+

$$\sigma_y^k \alpha_{ky} = p_-(y_- - \mu_y)^k + p_0(y_0 - \mu_y)^k + p_+(y_+ - \mu_y)^k \tag{24}$$

where the mean value and standard deviation of $y = y(x)$ are as follows:

$$\mu_y = p_- y_- + p_0 y_0 + p_+ y_+ \tag{25}$$

$$\sigma_y^2 = p_-[y_- - \mu_y]^2 + p_0[y_0 - \mu_y]^2 + p_+[y_+ - \mu_y]^2 \tag{26}$$

3.3. *Point estimates for a function of n variables*

The procedure leading to equations (17)–(26) has been generalized to a function of multiple variables $Z = G(\mathbf{X})$, where $\mathbf{X} = x_1, x_2, \ldots, x_n$. The joint probability density is assumed to be concentrated at points in the 2^n hyperquadrants of the space defined by $\mathbf{X}$. For a large n, the number of function recalls of $G(\mathbf{X})$ will be too large for practical applications. Rosenblueth[29] approximated $G(\mathbf{X})$ using the following function:

$$Z = G'(\mathbf{X}) = G^* \prod_{i=1}^{n} \left(\frac{Z_i}{G^*}\right) \tag{27}$$

and Idota *et al.*[31] approximated $G(\mathbf{X})$ using the following function:

$$Z = G'(\mathbf{X}) = \sum_{i=1}^{n} (Z_i - G^*) + G^* \tag{28}$$

where

$$Z_i = G(\mu_1, \ldots, \mu_{i-1}, x_i, \mu_{i+1}, \ldots, \mu_n) \tag{29}$$

$$G^* = G(\mu_1, \ldots, \mu_i, \ldots, \mu_n) \tag{30}$$

The mean value and standard deviation of approximation (27) are expressed as

$$\mu_Z = G^* \prod_{i=1}^{n} \left(\frac{\mu_{Zi}}{G^*}\right) \tag{31}$$

$$\sigma_Z^2 = \frac{\prod_{i=1}^{n}(\sigma_{Zi}^2 + \mu_{Zi}^2)}{(G^*)^{2(n-1)}} - \mu_Z^2 \tag{32}$$

The mean value and standard deviation of approximation (28) are expressed as

$$\mu_Z = \sum_{i=1}^{n} (\mu_{Zi} - G^*) + G^* \tag{33}$$

$$\sigma_Z^2 = \sum_{i=1}^{n} \sigma_{Zi}^2 \tag{34}$$

where μ_{Zi}, and σ_{Zi} are the mean value and standard deviation, respectively, of Z_i which are obtained using equations (25) and (26).

Using equations (27)–(34), only $2n + 1$ function calls of $G(\mathbf{X})$ are needed for computation of mean value and standard deviation of $G(\mathbf{X})$ having n random variables.

Substituting the functions $\mu_{\mathrm{m}}(\mathbf{X})$, $\mu^2_{\mathrm{m}}(\mathbf{X})$ and $\sigma^2_{\mathrm{m}}(\mathbf{X})$ in the previous section for $G(\mathbf{X})$ in equation (27) or equation (28), point estimates for $E[\mu_{\mathrm{m}}(\mathbf{X})]$, $E[\mu^2_{\mathrm{m}}(\mathbf{X})]$ and $E[\sigma^2_{\mathrm{m}}(\mathbf{X})]$ can be obtained, and the mean value μ_{M} and deviation σ_{M} of the maximum response can be evaluated.

3.4. *Investigation on the method of point estimates*

In the first example, consider the following function of a standard normal random variable:

$$y = \exp(u\zeta + \lambda) \tag{35}$$

where λ and ζ are parameters. In the present example, $\lambda = 1{\cdot}5$ and $\zeta = 0{\cdot}1$–$0{\cdot}6$ are assumed.

The variable y is a lognormal variable with parameter λ and ζ, and the first four moments are expressed exactly as

$$\mu_y = \exp\left(\lambda + \tfrac{1}{2}\zeta^2\right) \tag{36a}$$

$$\sigma_y^2 = \mu_y^2(\omega - 1) \tag{36b}$$

$$\alpha_3 = \sqrt{\omega - 1}\,(\omega + 2) \tag{36c}$$

$$\alpha_4 = \omega^4 + 2\omega^3 + 3\omega^2 - 3 \tag{36d}$$

where

$$\omega = \exp(\zeta^2), \tag{36e}$$

α_3 and α_4 are the third and fourth order dimensionless central moments, i.e. skewness and kurtosis of y, respectively.

The first four moments obtained using equations (24), (25) and (26) are depicted as (a), (b), (c) and (d) in Figure 1 along with the corresponding theoretical values. Figure 1 reveals that the mean value and standard deviation obtained by point estimates are in very good agreement with the exact values, but for third- and fourth-order moments, the results obtained by point estimates cannot be used as an approximation of the exact values.

In the second example, consider the following function of a lognormal normal random variable:

$$y = \ln(x) \tag{37}$$

where x is a lognormal variable with parameters λ and ξ. In the present example, $\lambda = 1{\cdot}5$ and $\zeta = 0{\cdot}1$–$0{\cdot}5$ are assumed.

The variable y is a normal random variable and the first four moments can be readily obtained as $\mu = \lambda$, $\sigma = \zeta$, $\alpha_3 = 0$, $\alpha_4 = 3$, exactly.

The first four moments obtained using equations (24), (25) and (26) are depicted as (a), (b), (c), and (d) in Figure 2 along with the corresponding theoretical values. Figure 2 shows that the μ and σ obtained using point estimates are in very good agreement with the exact values when ζ is assumed to be small (below 0·4). The accuracy of the results obtained using point estimates decreases as the order of the moments to be evaluated increases. For α_3 and α_4, the results obtained using point estimates cannot be used as an approximation of the exact values. Also note

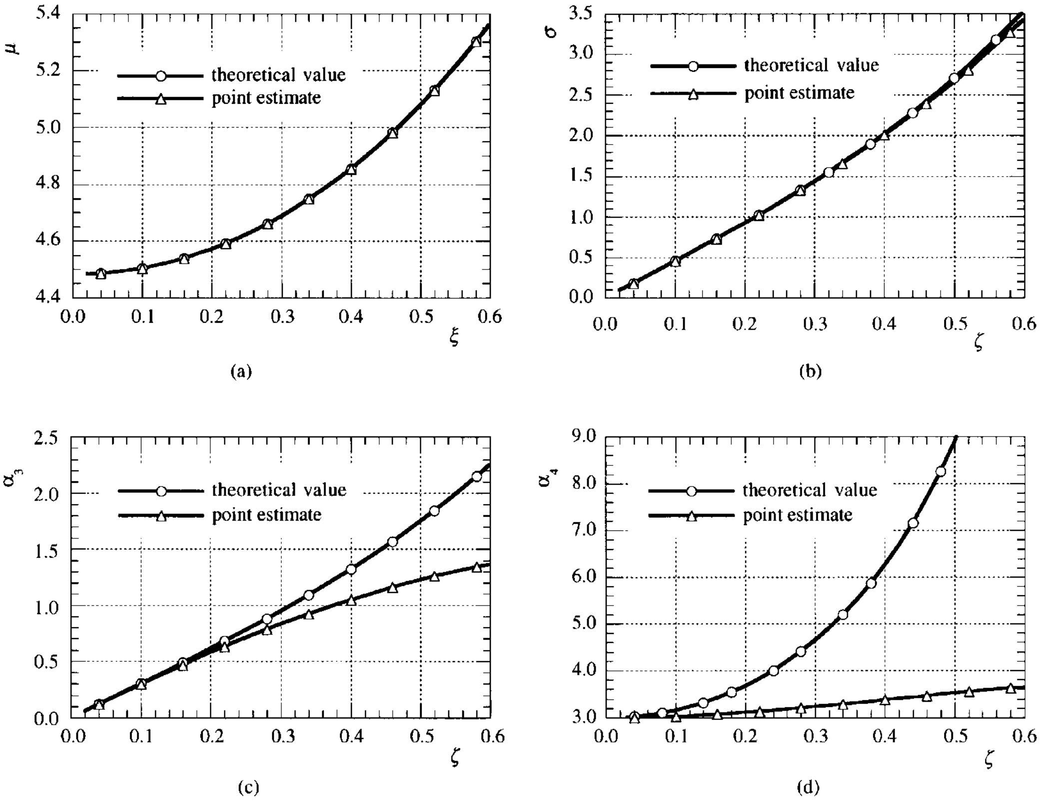

Figure 1. Point estimates for Example 1

that if the deviation of the random variable x is large, the estimating point x obtained using equation (20) becomes negative, and is therefore out of the definition area of the variable.

These two examples show that point estimates are only applicable for lower-order moments of functions of random variables with small deviation. Since the evaluation of response uncertainty described in Section 3.1 includes only the mean value of the functions $\mu_{\mathrm{m}}(\mathbf{X})$, $\mu^2_{\mathrm{m}}(\mathbf{X})$ and $\sigma^2_{\mathrm{m}}(\mathbf{X})$, point estimates are applicable in the present study.

4. TIME-VARIANT RELIABILITY ANALYSIS METHOD

4.1. Extensive first-order reliability method

In order to include the parameter uncertainties, one can first solve the problem with given parameters and then integrate over the system parameters to find the overall reliability, provided the probability information for the parameters is available. The integral is shown as

$$P_{\mathrm{F}} = \int_{\mathbf{X}} P_{\mathrm{f}}(\mathbf{X}) f(\mathbf{X})\, \mathrm{d}\mathbf{X} \tag{38}$$

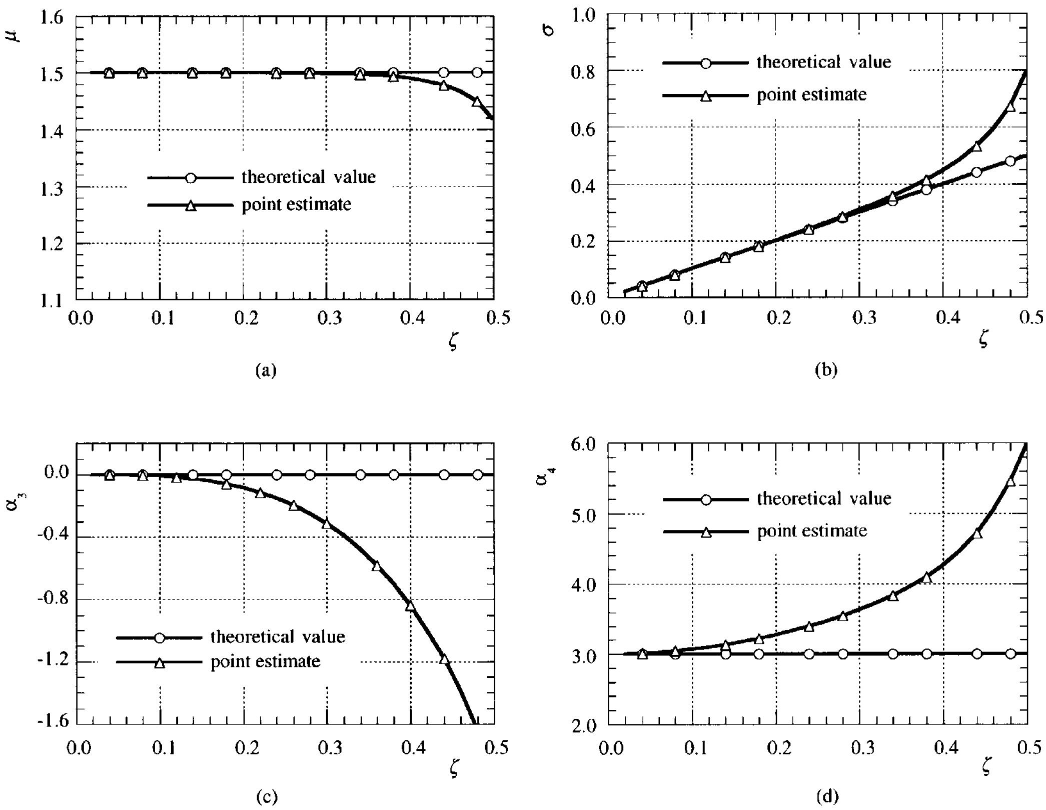

Figure 2. Point estimates for Example 2

where $P_f(\mathbf{X})$ is the conditional failure probability for a given $\mathbf{X}$, which is evaluated using state-of-the-art techniques, and $f(\mathbf{X})$ is the joint probability density function of $\mathbf{X}$.

However, the computation could become excessive since repeated solutions of stochastic structural response are required, and so an approximate method having good accuracy and numerical efficiency is needed. Therefore, Wen and Chen[4] developed the following performance function in standard normal space, in which only one auxiliary random variable is introduced.

$$G(\mathbf{X}, u_{n+1}) = u_{n+1} - \Phi^{-1}[P_f(\mathbf{X})] \tag{39}$$

The gradients of the performance function with respect to $\mathbf{U}$ are given by

$$\left\{\frac{\partial G}{\partial u_i}\right\}_{i \leqslant n} = -[J_n^{-1}]^{\mathrm{T}} \left\{\frac{\partial [P_f(X)]}{\partial x_i}\right\}_{i \leqslant n} \tag{40}$$

where $\mathbf{U}$ is the standard normal variable vector transformed from $\mathbf{X}$ by Rosenblatt transformation, and u_{n+1} is a standard normal random variable independent of $\mathbf{U}$. $[J]$ is the Jacobian matrix for the transformation of variables.

This performance function enables an evaluation of the dynamic structural reliability that includes parameter uncertainties to be performed using FORM and is referred to as Extensive FORM (EFORM) in the present paper. Calculation of the gradients of the performance function is an important step of the EFORM. However, it is not always easy to obtain these gradients,[18] especially for the case where non-linearity of the structural performance is considered. In order to avoid such difficulties in obtaining the gradients, in this paper the performance function is approximated using Response Surface Approach (RSA).[19]

4.2. *Response surface approach in standard space*

RSA is a statistical analysis method that examines the relationship between experimental response and variations in the values of input variables. The basic concept of RSA is to replace the original implicit performance function by an approximated explicit function (generally a second-order polynomial) expressed in terms of basic random variables.[32–34] For time-variant reliability analysis, Yao and Wen[19] have introduced RSA to avoid the sensitivity analysis required in EFORM. However, because the response surface function is expressed as a polynomial of basic random variables in original space, when the computational accuracy of failure probability is improved by using SORM, it is necessary to compute the Hessian matrix, and to carry out its rotational transformation and eigenvalue analysis. Alternatively, one can use the point-fitting SORM approximation[35] after the response surface has been obtained, but additional iteration effort will be needed.[36] In order to improve on this weakness, in this paper the performance function is directly approximated in standard normal space. This has the following two advantages:

(1) Because the response surface function is expressed directly as a second-order polynomial of standard normal random variables, it is very simple to obtain the second-order derivative matrix (the scaled Hessian matrix) and compute the failure probability using the simple approximation of SORM.[35,37]
(2) In RSA, fitting points are controlled by their distance from the original point, which is generally expressed as a multiple of the standard deviation of the random variable. If the response surface function is expressed as the function of standard normal random variables, the factors become very simple. This is because all the standard deviations of all the standard normal random variables are equal to 1.

For simplification, the performance function shown in equation (39) is approximated by the following second-order polynomial of standard normal random variables, in which the mixed terms are neglected

$$G'(\mathbf{U}) = a_0 + u_{n+1} + \sum_{j=1}^{n} \gamma_j u_j + \sum_{j=1}^{n} \lambda_j u_j^2 \tag{41}$$

where a_0, γ_j, and λ_j are $2n+1$ regression coefficients with j ranging from 1 to n.

If the practical performance function $G(\mathbf{U})$ in equation (39) is fitted by $G'(\mathbf{U})$ of equation (41) at the fitting points in the vicinity of the design point, the regression coefficients a_0, γ_j, and λ_j can be determined from the linear equations of a_0, γ_j, and λ_j obtained at each fitting point.

For the obtained design point $\mathbf{U}_c$ in standard normal space, fitting points along the co-ordinate axes are selected. Along each axis $u_j, j = 1, \ldots, n$, two points having the co-ordinates $(\mathbf{U}'_c, u_{cj} - \delta)$

and $(\mathbf{U}'_c, u_{cj} + \delta)$ are selected, where $\mathbf{U}'_c = \{u_{ck}, k = 1, \ldots, n \text{ except } j\}$ represents the co-ordinates of the design point along all the axes except the j-axis, and δ is a factor which represents the distance from the central point to the fitting point. Transform the fitting points into original space using Rosenblatt transformation, and fit the original performance function by the performance function approximation in equation (41) at these points. The regression coefficients in equation (41) can now be obtained.

Because the design point is not generally known beforehand, this paper uses the iterative RSA procedure in the point-fitting SORM[35] to approach the performance function in the course of obtaining the design point. Using the procedure, only $m(2n + 1)$ repetitions of nonlinear random vibration analysis are required for evaluation of time variant reliability with n random variables, where m is the number of iterations. As shown in the ensuing sections of the present paper, this procedure is reasonably accurate.

4.3. *Computation of failure probability*

After obtaining the response surface function shown in equation (41), the first-order reliability index and its corresponding failure probability can be readily obtained. For the case of a strong non-linear performance function, in order to improve the analytical accuracy, the failure probability can be obtained using MCS or SORM. In this paper, the failure probability is computed using the empirical SORM reliability index.[37]

Because the response surface function has the same form as the point-fitted performance function,[35] the scale Hessian matrix corresponding to equation (41) is readily obtained as

$$\mathbf{B} = \frac{2}{|\nabla G'|} \begin{bmatrix} \lambda_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda_n \end{bmatrix} \tag{42}$$

where

$$|\nabla G'| = \sqrt{1 + \sum_{j=1}^{n} (\gamma_j + 2\lambda_j u_j^*)} \tag{43}$$

The sum of the principle curvatures and the average principle curvature radius of the limit state surface at design point $\mathbf{U}^*$ can be expressed as:[35]

$$K_S = \frac{2}{|\nabla G'|} \sum_{j=1}^{n} \lambda_j \left[1 - \frac{1}{|\nabla G'|^2} (\gamma_j + 2\lambda_j u_j^*)^2 \right] \tag{44}$$

$$R = \frac{n-1}{K_s} \tag{45}$$

With the aid of K_s and R expressed in equations (44) and (45), the failure probability corresponding to the point-fitted performance function (response surface function) can be obtained by substituting equations (44) and (45) in the following empirical second-order reliability index, which is in closed form:

$$\beta_s = -\Phi^{-1} \left[\Phi(-\beta_F) \left(1 + \frac{\phi(\beta_F)}{R\Phi(-\beta_F)} \right)^{-(n-1)/2(1 + 2K_s/10(1 + 2\beta_F))} \right], \quad K_s \geqslant 0 \tag{46}$$

$$\beta_s = \left(1 + \frac{2{\cdot}5K_s}{2n - 5R + 25(23 - 5\beta_F)/R^2}\right)\beta_F + \frac{1}{2}K_s\left(1 + \frac{K_s}{40}\right), \quad K_s < 0 \tag{47}$$

where K_s is the sum of the principle curvatures of the limit state surface given as in equation (44), R the average principle curvature radius given as in equation (45), n the number of random variables, β_F the first-order reliability index and β_s the second-order reliability index.

4.4. *Inclusion of random variables having no CDF*

In the analysis method described above, all random variables are assumed to be continuous variables having a known CDF. However, the CDF of some of these variables may not be actually obtainable, and their probability information may be expressed only as cumulate moments. In order to include these random variables, the High-Order Moment Standardization Technique (HOMST)[38,39] is used.

When the CDF of a random variable cannot be obtained, the histogram is assumed to be obtainable from statistical data. The failure probability of equation (38) can be expressed in sum form:

$$P_F = \sum_i P_f(x_i)h(x_i)\,\Delta x_i \tag{48}$$

where $P_f(x_i)$ is the conditional failure probability when the random variable takes the given value of $x = x_i$, and $h(x_i)$ is the value of the histogram when x is equal to x_i.

Using HOMST, the random variable can be transformed into a standard normal random variable by the high-order moment standardization function S,

$$y = \frac{x - \mu_x}{\sigma_x} \tag{49}$$

$$u = \frac{1}{a}\left[\alpha_{3y} + 3(\alpha_{4y} - 1)y - \alpha_{3y}y^2\right] \tag{50}$$

$$a = \sqrt{(5\alpha_{3y}^2 - 9\alpha_{4y} + 9)(1 - \alpha_{4y})} \tag{51}$$

where α_{ky} is the kth-order dimensionless central moment of y, which is equal to the kth-order dimensionless central moment of x according to the definition of high-order moment, u is the standard normal random variable.

Because u is a continuous random variable, a continuous random variable x' can be obtained by the inverse transformation S^{-1} corresponding to u.

$$x' = S^{-1}(u) \tag{52}$$

Although x and x' are different random variables, they correspond to the same standard normal random variable and have the same high-order moments and the same statistical information source. Therefore, $f(x')$ can be considered to be an anticipated continuous distribution of x. Using this continuous distribution, the failure probability shown in equation (48) can again be expressed as in equation (38), and reliability analysis can be performed using the EFORM equations described in the previous section. The Jacobinan matrix can be obtained directly from equations (49), (50) and (51), instead of from the Rosenblatt transformation:

$$J_{ii} = \frac{\partial u_i}{\partial x_i} = \frac{1}{a\sigma_x}\left[3(\sigma_{4y} - 1) - 2\alpha_{3y}y\right] \tag{53}$$

and the inverse function of S is expressed as

$$x' = \frac{\sigma_x}{2\alpha_3}\left[3(\alpha_4 - 1) - \sqrt{9(\alpha_4 - 1)^2 + 4\alpha_3(\alpha_3 - ua)}\right] + \mu_x \tag{54}$$

When both random variables having continuous CDF and those having no CDF are considered, the random variables can be divided into two groups $\mathbf{X} = [\mathbf{X}_1, \mathbf{X}_2]$, where $\mathbf{X}_1$ are the random variables having CDF, and $\mathbf{X}_2$ those having no CDF. For $\mathbf{X}_2$, using the anticipated distribution $f_2(\mathbf{X}_2')$ obtained from HOMST described above for $\mathbf{X}_2$, the total failure probability can be written as

$$P_{\mathrm{F}} = \int_{\mathbf{X}_1, \mathbf{X}_2'} P_{\mathrm{f}}(\mathbf{X}_1, \mathbf{X}_2') f_1(\mathbf{X}_1) f_1(\mathbf{X}_2')\, \mathrm{d}\mathbf{X}_1\, \mathrm{d}\mathbf{X}_2' \tag{55}$$

Equation (55) can be solved using the EFORM combined with the RSA in standard normal space described in the previous section. When doing this, the Jacobian matrix of $\mathbf{X}_1$ is obtained by Rosenblatt transformation, and that of $\mathbf{X}_2$ is obtained using HOMST. In this way, the random variables without CDF can be included with almost no extra effort. Note that the histogram $h(x_i)$ and the anticipated distribution $f_2(\mathbf{X}_2')$ of $\mathbf{X}_2$ are not needed in the computation; they are used here only for convenience of description.

5. NUMERICAL EXAMPLE AND INVESTIGATIONS

5.1. Structural model and conditional random vibration analysis

In this section, the response uncertainty and time-variant reliability analysis for a 15F steel structure having a total height of 52·5 m is performed. The height, weight, initial stiffness and strength of each floor are listed in Table I. The structural model is assumed to be a shearing hysteretic MDF model, and the force–deformation relationship is assumed to be bilinear. The damping and stiffness ratios are assumed to be 0·02 and 0·05, respectively.

The acceleration spectrum recommended by the Architecture Institute of Japan (AIJ)[40] is used as the earthquake input and is expressed as

$$S_A(T, h) = \begin{cases} \left(1 + \dfrac{f_A - 1}{d}\right) F_h G_A R_A A_0 & 0 \leqslant T \leqslant dT_{\mathrm{c}} \\ F_h G_A R_A A_0 & dT_{\mathrm{c}} \leqslant T \leqslant T_{\mathrm{c}} \\ \dfrac{2\pi F_h G_V R_V V_0}{T} & T_{\mathrm{c}} \leqslant T \end{cases} \tag{56}$$

where f_A, f_V, d, G_A, G_V, and T_C are parameters of the response spectrum and are assumed to be 2·5, 2·5, 0·5, 1·2, 2·0, and 0·55, respectively, according to the recommendations of the AIJ.[40]

As the input of the peak ground acceleration, the maximum value of 818 gal, obtained on January 17th, 1995 during the Hanshin-Awaji earthquake, is used. The computational results using deterministic structural parameters are listed in Table II, in which $R_{\max}$ represents the maximum response of relative storey displacement. Table II shows that all floors enter the ductile area when subjected to the previously mentioned input. The maximum ductility ratio occurs at

Table I. Structural parameters

Floor	H (cm)	W (t)	K (t/cm)	Q (t)	K_1/K_0
15	400	397	404	490	0·05
14	400	1278	615	1909	0·05
13	335	1909	1084	3044	0·05
12	310	1400	1429	3739	0·05
11	310	1386	1687	4216	0·05
10	310	1465	1979	4464	0·05
9	310	1624	2200	5271	0·05
8	310	1595	2488	5665	0·05
7	310	1615	2889	6141	0·05
6	310	1961	3347	5669	0·05
5	450	3546	2999	5953	0·05
4	450	4613	2909	7224	0·05
3	500	5802	2803	8735	0·05
2	400	7593	3625	9922	0·05
1	475	6007	4507	10330	0·05

H: Floor height W: Floor weight
K: Initial stiffness Q: Initial strength
K_1/K_0: Stiffness ratio

Table II. Stochastic response using deterministic parameters

Floor	σ_r	μ of R_{max}	Duct. ratio	σ of R_{max}	P_f	β
15F	0·877	2·30	1·90	0·46	0·0	×
14F	2·293	6·16	1·98	1·21	0·0	×
13F	2·892	7·62	2·71	1·57	0·0	×
12F	2·880	7·47	2·85	1·59	0·0	×
11F	2·988	7·52	3·01	1·68	$1{\cdot}2\times10^{-7}$	5·60
10F	3·100	7·68	3·40	1·77	$1{\cdot}0\times10^{-5}$	4·27
9F	2·944	7·25	3·03	1·71	$2{\cdot}9\times10^{-7}$	5·00
8F	2·894	7·09	3·11	1·69	$9{\cdot}5\times10^{-7}$	4·76
7F	2·655	6·52	3·07	1·50	$6{\cdot}5\times10^{-7}$	4·84
6F	2·917	7·16	4·23	1·72	$2{\cdot}0\times10^{-3}$	2·88
5F	3·947	9·60	4·84	2·34	$1{\cdot}7\times10^{-2}$	2·12
4F	4·276	10·26	4·13	2·55	$1{\cdot}5\times10^{-3}$	2·97
3F	4·662	11·22	3·60	2·77	$8{\cdot}0\times10^{-5}$	3·77
2F	4·168	10·16	3·71	2·44	$1{\cdot}3\times10^{-4}$	3·65
1F	3·599	8·97	3·91	2·05	$3{\cdot}2\times10^{-4}$	3·41

the fifth floor, and the corresponding mean value of maximum response is found to be 9·60 with a standard deviation of 2·34.

Assuming the excursion level is taken to be 7 based on the deformation capacity of buildings in seismic design recommended by the AIJ,[41] the computational results of failure probability with given parameters are listed in Table II. The conditional failure probability corresponding to the fifth floor is found to be 0·017.

5.2. *Parameter uncertainties*

Seven parameters, four structural parameters and three excitation characteristics, have been considered. The uncertainty of structural parameters, the initial stiffness K_0, initial strength Q_0, floor weight W and damping ratio ξ in each floor are assumed to be totally correlated. Each parameter is multiplied by a random variable having a unit mean value

$$K'_0 = x_K K_0 \tag{57a}$$

$$Q'_0 = x_Q Q_0 \tag{57b}$$

$$W' = x_W W \tag{57c}$$

$$\xi' = x_\xi \xi \tag{57d}$$

$$D' = x_D D \tag{57e}$$

$$S'_A = x_S S_A \tag{57f}$$

where $x_K, x_Q, x_W, x_\xi, x_D$ and x_S are random variables having a mean value of 1. K_0, Q_0, W, ξ, D and S_A are the deterministic values of initial stiffness, yield strength, floor weight, damping ratio, duration and response spectra, respectively, and K'_0, Q'_0, W', ξ', D' and S'_A are the corresponding variables that include uncertainties.

According to References 4, 28, 38 and 41, $x_K, x_Q, x_W, x_\xi, \mathrm{x}_D$ are assumed to be lognormal random variables with coefficients of variation of 0·1, 0·2, 0·1, 0·4 and 0·3 respectively. The first four moments of the standard response spectrum are taken to be $\mu = 1{\cdot}0$. $\sigma = 0{\cdot}26$, $\alpha_3 = 0{\cdot}798$, $\alpha_4 = 7{\cdot}152$ according to Reference 28.

The distribution of the peak ground acceleration a_g is obtained directly from th excursion probability curve of the annual peak ground acceleration under the assumption that the earthquake occurs in accordance with the Poisson's Law:

$$f_A(a) = \left(\frac{\alpha}{a_0}\right)\left(\frac{a}{a_0}\right)^{-(\alpha+1)} \tag{58}$$

where a_0 is the minimum value of a_g, which is taken to be 20 gal, and α is taken to be 2·34.

5.3. *Response uncertainty evaluation*

In order to investigate the evaluation procedure for response uncertainty, the uncertainties of only four parameters, i.e. x_K, x_Q, x_W and x_ξ are considered. All four random variables have known CDF, so the results can be confirmed using the Monte-Carlo Simulation (MCS). Evaluations are conducted using the approximation functions given in equations (27) and (28). The standard deviations of the maximum response obtained using point estimates are listed in Table III along with those obtained using MCS with a sample size of 10 000. Table III shows that although only $2n + 1 = 9$ repetitions of nonlinear random vibration analysis are required in equations (27) and (28), both of these approximation functions yield good results. The largest error for equation (27) is 1·815 per cent and that for equation (28) is 2·383 per cent. That is to say, the results obtained using point estimates using either equation (27) or equation (28) can be used as an accurate approximation of the actual values. Since the errors obtained using equation (27) are generally smaller than those obtained using equation (28), the present paper will use equation (27) for the

Table III. Comparison of MCS and PEM with four parameter uncertainties (p of maximum response)

Floor	MCS	Point estimates			
		Equation (27)	Error (%)	Equation (28)	Error (%)
1	2·3523	2·3096	1·815	2·2962	2·383
2	2·7234	2·6936	1·094	2·6839	1·451
3	3·0926	3·0475	1·458	3·0319	1·964
4	2·8554	2·8083	1·649	2·7977	2·020
5	2·6448	2·6237	0·798	2·6154	1·113
6	1·8958	1·8829	0·680	1·8842	0·610
7	1·6597	1·6535	0·373	1·6561	0·215
8	1·7935	1·7935	0·000	1·7966	0·173
9	1·8132	1·8235	0·568	1·8258	0·695
10	1·9009	1·9171	0·852	1·9194	0·972
11	1·7939	1·8113	0·970	1·8150	1·174
12	1·7015	1·7193	1·046	1·7229	1·258
13	1·6691	1·6732	0·241	1·6689	0·013
14	1·3607	1·3712	0·772	1·3765	1·162
15	0·5303	0·5378	1·414	0·5390	1·646

evaluation of the uncertainties of the maximum response in which all the parameter uncertainties are considered.

In order to investigate the effect of the parameter uncertainties, the mean value of the maximum response using deterministic parameters, two parameter uncertainties (x_K and x_Q only), four parameter uncertainties (x_K, x_Q, x_W and x_ξ only) and six parameter uncertainties (x_K, x_W, x_ξ, x_D and x_S) are depicted in Figure 3, in which the inclusion of parameter uncertainties is revealed to make the mean values of the maximum response larger for some floors and smaller for others. Generally, the inclusion of the parameter uncertainties have little effect on the mean value of the maximum response.

The standard deviation of the maximum response corresponding to Figure 3 are depicted in Figure 4, in which the deviations including parameter uncertainties are very different from those using deterministic parameters. From the comparison between Figures 3 and 4, one can see that the parameter uncertainties have a much greater effect on the standard deviation than on the mean value of the maximum response. Figure 4 reveals that the greater the number of parameter uncertainties included, the larger the standard deviation of the maximum response. Since the standard deviation using six parameter uncertainties is much larger than that using deterministic parameters, two parameter uncertainties and four parameter uncertainties, the effect of the uncertainties included in the excitation properties is found to be dominant on the uncertainties of the maximum response.

5.4. *Time-variant reliability analysis*

In order to investigate the efficiency of the procedure proposed for time-variant reliability analysis, uncertainties of only two parameters, i.e. x_K and x_Q, are first considered. The fitting points in standard normal space are controlled by $\delta = 0{\cdot}5$.

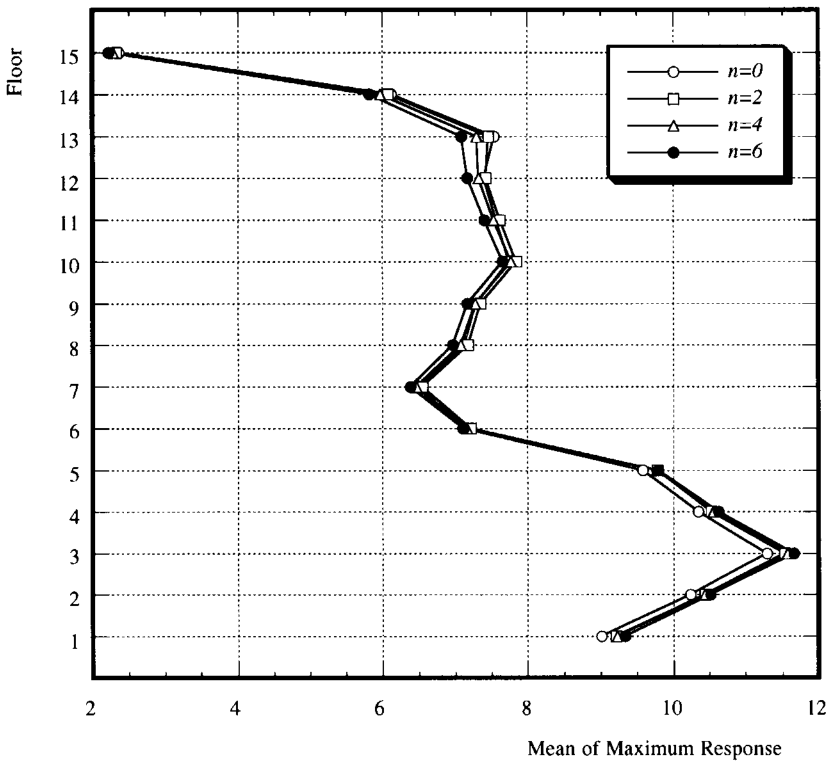

Figure 3. Mean values of maximum response including parameter uncertainties

For the fifth floor, convergence is reached easily after five iterations. The first-order reliability index is obtained as $\beta_F = 1{\cdot}118$, with corresponding failure probability of $P_F = 0{\cdot}1319$. Using the SORM approximation described in Section 4·3, the sum of the principle curvatures of the limit state surface is readily obtained as $K_s = 0{\cdot}0689$, with a corresponding average principle curvature radius of $R = 29{\cdot}042$. It can be seen from this that the non-linearity of the performance function is very weak. With the aid of K_s and R, the second-order reliability index is obtained as $\beta_s = 1{\cdot}151$, with corresponding failure probability of $P_s = 0{\cdot}1249$. For the purpose of comparison, MCS with 50 000 trials is also conducted and the failure probability is obtained as 0·1236. One can see that the SORM result obtained in this paper agrees with the MCS result quite well.

In order to investigate the procedure of the RSA, the response surface obtained after the first, third and fifth iterations are depicted in Figures 5–7, in which the black point is the design point of the limit state surface and the dashed lines describe the coordinates of the design point. In these figures, the surface with rough mesh is the response surface described by equation (41) and that with fine mesh is the true limit state surface described by equation (39). From Figures 5–7, one can see that although the response surface obtained after the first iteration does not approximate the true limit surface in the vicinity of the design point, as the number of iterations increases, the approximation gradually becomes better. The response surfaces obtained after the fifth iteration agrees with the limit state surface quite well in the vicinity of the design point.

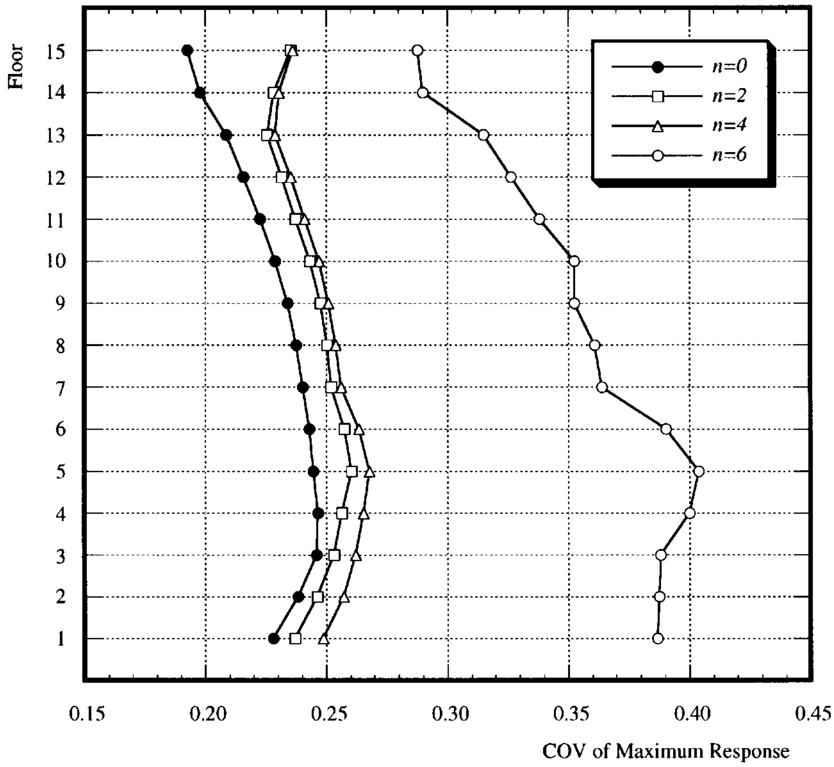

Figure 4. COV of maximum response including parameter uncertainties

For the 1st to 6th floors, convergence can be reached easily in 3–6 iterations. For the 12th to 15th floors, convergence cannot be reached, because the EFORM is based on the computation of $P_f(\mathbf{X})$ and the algorithm of FORM, in which, when $P_f(\mathbf{X})$ becomes 0, the inverse function of the normal distribution becomes infinite. Because conditional failure probabilities for the 7th to 11th floors are too small (approximately 10^{-7} as listed in Table II), and the accurate inverse function of the normal distribution becomes difficult to obtain, the computation procedure becomes unstable. Because of this, the results are obtained by adjusting δ to a value of 0·5–0·7. The FORM/SORM results for each floor are listed in Table IV, along with those obtained by using MCS with 50 000 trials for the sake of comparison. From Table IV, the failure probabilities when parameter uncertainties are included can clearly be seen to be much larger than those when only deterministic parameters are used. There is good agreement between the results obtained using the proposed procedure and those obtained using MCS. One can also see that the difference between the reliability index with given parameters and that with parameter uncertainties increases as the reliability index increases.

To investigate the efficiency of including the random variables that have no CDF, analysis is also conducted using the high-order moments. The high-order moments corresponding to the initial stiffness x_K are obtained as $\alpha_3 = 0{\cdot}301$, $\alpha_4 = 3{\cdot}1615$, and those corresponding to the initial strength x_Q as $\alpha_3 = 0{\cdot}608$, $\alpha_4 = 3{\cdot}6644$. The reliability results obtained using high-order moments instead of CDF are listed in Table V. From Table V, one can see that the computational results

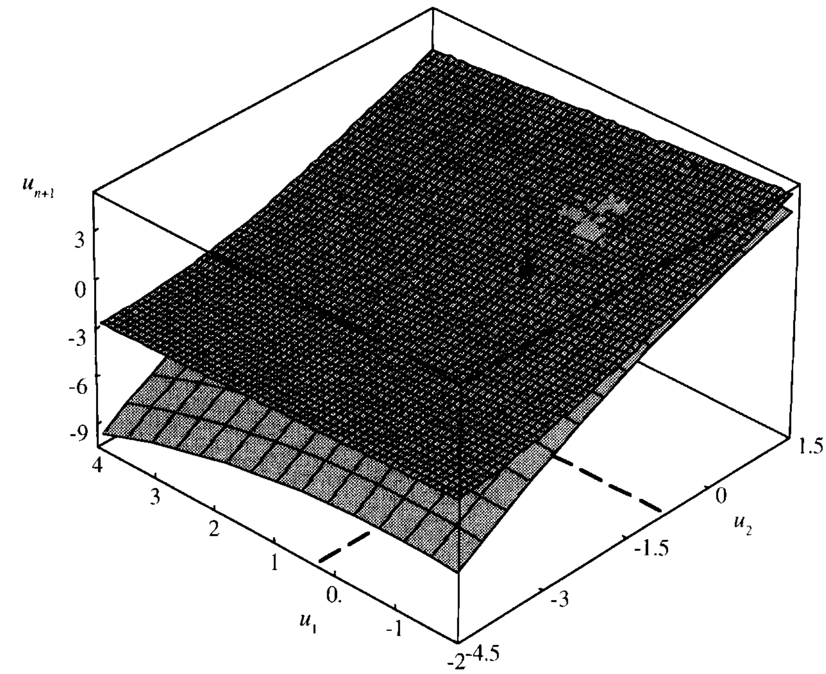

Figure 5. Response surface obtained after first iteration

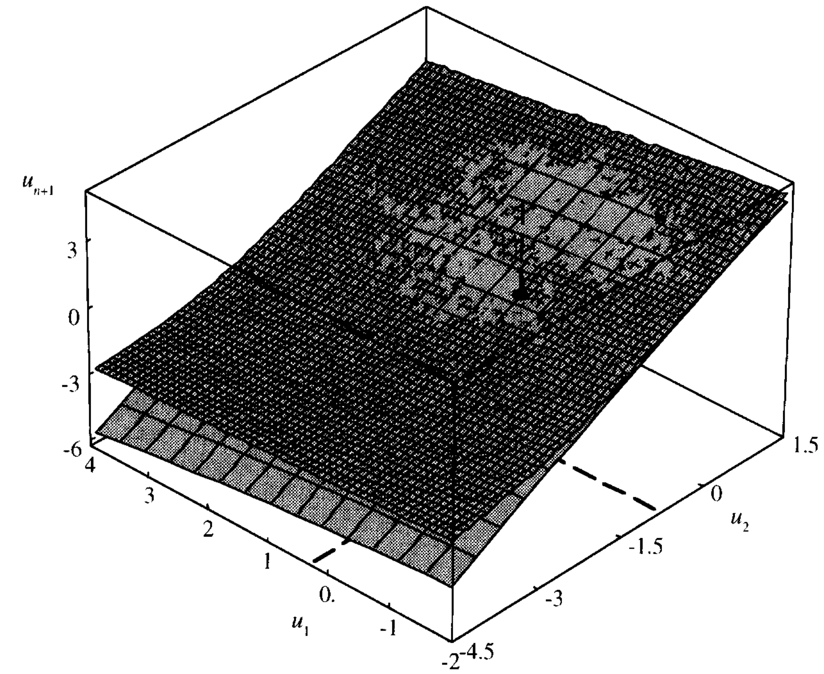

Figure 6. Response surface obtained after third iteration

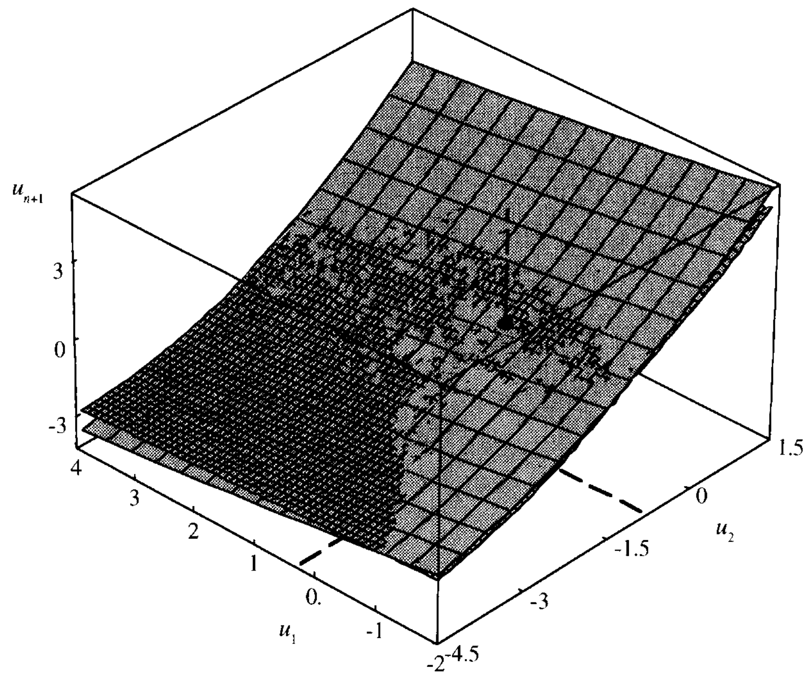

Figure 7. Response surface obtained after fifth iteration

Table IV. Results with two parameter uncertainties using known CDF

Floor no.	Present method						MCS	
	FORM		SORM		δ	No. of iter.	(50 000)	
	β_F	P_F	β_s	P_F			β	P_F
15	×	×	×	×	×	×	×	0
14	×	×	×	×	×	×	×	0
13	×	×	×	×	×	×	3·66	$1{\cdot}3 \times 10^{-4}$
12	×	×	×	×	×	×	3·28	$5{\cdot}3 \times 10^{-4}$
11	2·797	$2{\cdot}58 \times 10^{-3}$	2·876	$2{\cdot}02 \times 10^{-2}$	0·7	4	2·93	$1{\cdot}7 \times 10^{-3}$
10	2·292	$1{\cdot}09 \times 10^{-2}$	2·453	$7{\cdot}08 \times 10^{-3}$	0·7	8	2·40	$8{\cdot}3 \times 10^{-3}$
9	2·729	$3{\cdot}18 \times 10^{-3}$	2·898	$1{\cdot}88 \times 10^{-3}$	0·6	3	2·83	$2{\cdot}4 \times 10^{-3}$
8	2·606	$4{\cdot}58 \times 10^{-3}$	2·649	$4{\cdot}03 \times 10^{-3}$	0·7	5	2·71	$3{\cdot}4 \times 10^{-3}$
7	2·676	$3{\cdot}73 \times 10^{-3}$	2·792	$2{\cdot}62 \times 10^{-2}$	0·7	3	2·77	$2{\cdot}8 \times 10^{-3}$
6	1·569	$5{\cdot}83 \times 10^{-2}$	1·651	$4{\cdot}94 \times 10^{-2}$	0·5	3	1·63	$5{\cdot}2 \times 10^{-2}$
5	1·118	0·1319	1·151	0·1249	0·5	5	1·16	0·1236
4	1·561	$4{\cdot}93 \times 10^{-2}$	1·574	$5{\cdot}77 \times 10^{-2}$	0·5	6	1·62	$5{\cdot}2 \times 10^{-2}$
3	2·005	$2{\cdot}25 \times 10^{-2}$	2·024	$2{\cdot}15 \times 10^{-2}$	0·5	3	2·07	$1{\cdot}9 \times 10^{-2}$
2	1·968	$2{\cdot}45 \times 10^{-2}$	1·965	$2{\cdot}47 \times 10^{-2}$	0·5	6	2·00	$2{\cdot}3 \times 10^{-2}$
1	1·744	$4{\cdot}06 \times 10^{-2}$	1·791	$3{\cdot}66 \times 10^{-2}$	0·5	4	1·82	$3{\cdot}4 \times 10^{-2}$

Table V. Results with two parameter uncertainties using high-order moments

Floor no.	FORM		SORM		δ	No. of iter.
	β_F	P_F	β_s	P_F		
15	×	×	×	×	×	×
14	×	×	×	×	×	×
13	×	×	×	×	×	×
12	×	×	×	×	×	×
11	2·724	$3{\cdot}23\times10^{-3}$	2·800	$2{\cdot}56\times10^{-2}$	0·7	6
10	2·284	$1{\cdot}12\times10^{-2}$	2·340	$9{\cdot}64\times10^{-3}$	0·7	7
9	2·638	$4{\cdot}17\times10^{-3}$	2·620	$4{\cdot}39\times10^{-3}$	0·8	3
8	2·554	$5{\cdot}32\times10^{-3}$	2·621	$4{\cdot}39\times10^{-3}$	0·6	12
7	2·634	$4{\cdot}23\times10^{-3}$	2·792	$2{\cdot}62\times10^{-2}$	0·6	4
6	1·575	$5{\cdot}76\times10^{-2}$	1·616	$5{\cdot}31\times10^{-2}$	0·5	3
5	1·120	0·1313	1·167	0·1217	0·5	6
4	1·571	$5{\cdot}81\times10^{-2}$	1·554	$6{\cdot}02\times10^{-2}$	0·5	6
3	2·029	$2{\cdot}12\times10^{-2}$	1·987	$2{\cdot}35\times10^{-2}$	0·5	6
2	1·959	$2{\cdot}50\times10^{-2}$	1·990	$2{\cdot}33\times10^{-2}$	0·5	4
1	1·761	$3{\cdot}91\times10^{-2}$	1·781	$3{\cdot}74\times10^{-2}$	0·7	3

obtained using high-order moments agree approximately with those obtained using CDF. In other words, the procedure of including the random variables that have no CDF is effective.

In the case of inclusion of all of the seven parameter uncertainties considered in the present paper, i.e. x_K, x_Q, x_W, x_ξ, x_D, x_S and a_g, the fitting points in standard normal space are controlled by $\delta = 0{\cdot}6$, for all the random variables except a_g, for which a value of $\delta = 0{\cdot}08$ is used. For the 5th floor, convergence is reached after only three iterations and the response surface function is obtained as

$$\begin{aligned} G' = 24{\cdot}086 + u_{n+1} - 0{\cdot}228u_1 + 1{\cdot}556u_2 - 0{\cdot}358u_3 - 0{\cdot}0229u_4 \\ - 2{\cdot}055\times10^{-4}u_5 - 1{\cdot}707u_6 - 1{\cdot}400u_7 - 0{\cdot}0519u_1^2 + 0{\cdot}202u_2^2 \\ - 0{\cdot}0104u_3^2 - 0{\cdot}0805u_4^2 - 0{\cdot}0141u_5^2 + 0{\cdot}202u_6^2 - 1{\cdot}327u_7^2 \end{aligned} \tag{59}$$

where $u_1, u_2, u_3, u_4, u_5, u_6$ and u_7 are standard normal random variables corresponding to K_0, Q_0, W, ζ, S_A, D and a_g, respectively.

Using equation (59), the first-order reliability index is obtained as $\beta_F = 3{\cdot}679$, with corresponding failure probability of $P_F = 1{\cdot}17\times10^{-4}$. The average principle curvature and curvature radius corresponding to equation (59) are readily obtained as $K_S = 3{\cdot}276\times10^{-2}$ and $R = 213{\cdot}685$. The second-order reliability index is obtained as $\beta_S = 3{\cdot}695$, with corresponding failure probability of $P_F = 1{\cdot}10\times10^{-4}$. The SORM results only improved those of FORM slightly because the non-linearity in equation (59) is not strong.

For the 1st to 10th floors, the convergency is reached quickly after 3–5 iterations; for the 11th to 15th floors, convergence is not reached because the conditional failure probabilities $P_f(\mathbf{X})$ are too small. The design points and FORM/SORM results obtained from the reliability evaluation procedure are listed in Table VI, where the design points have been transformed to original apace for convenience of comparison. From Table VI, the values at the design point corresponding to the random variables that have relatively larger uncertainty, i.e. the yield strength x_Q, the

Table VI. Computational results with seven parameter uncertainties

Floor no.	Design point							FORM		SORM		Iter.
	x_K	x_Q	x_W	ξ	D	S_A	a	β_F	P_F	β_S	P_F	
10	1·008	0·891	1·016	0·925	0·955	1·134	1·081	3·857	$5{\cdot}74\times10^{-5}$	3·997	$3{\cdot}21\times10^{-5}$	5
9	1·003	0·908	1·006	0·921	0·954	1·096	1·346	3·935	$4{\cdot}15\times10^{-5}$	4·056	$2{\cdot}49\times10^{-5}$	5
8	1·001	0·890	1·011	0·939	0·958	1·141	1·214	3·923	$4{\cdot}37\times10^{-5}$	3·998	$3{\cdot}20\times10^{-5}$	4
7	1·006	0·903	1·007	0·929	0·959	1·119	1·272	3·924	$4{\cdot}35\times10^{-5}$	4·021	$2{\cdot}90\times10^{-5}$	4
6	1·006	0·906	1·006	0·930	0·959	1·123	0·939	3·746	$8{\cdot}99\times10^{-5}$	3·853	$5{\cdot}82\times10^{-5}$	3
5	1·003	0·897	1·006	0·931	0·961	1·116	0·839	3·679	$1{\cdot}17\times10^{-4}$	3·695	$1{\cdot}10\times10^{-5}$	3
4	1·005	0·894	1·008	0·928	0·955	1·127	0·923	3·749	$8{\cdot}87\times10^{-5}$	3·779	$7{\cdot}89\times10^{-5}$	4
3	1·005	0·898	1·006	0·924	0·955	1·121	1·050	3·815	$6{\cdot}80\times10^{-5}$	3·901	$4{\cdot}78\times10^{-5}$	5
2	1·005	0·897	1·007	0·918	0·952	1·124	1·018	3·798	$7{\cdot}28\times10^{-5}$	3·876	$5{\cdot}30\times10^{-5}$	4
1	1·005	0·905	1·007	0·945	0·955	1·112	0·975	3·778	$7{\cdot}92\times10^{-5}$	3·706	$1{\cdot}05\times10^{-4}$	3
Init.	1·000	1·000	1·000	1·000	1·000	1·000	0·818					

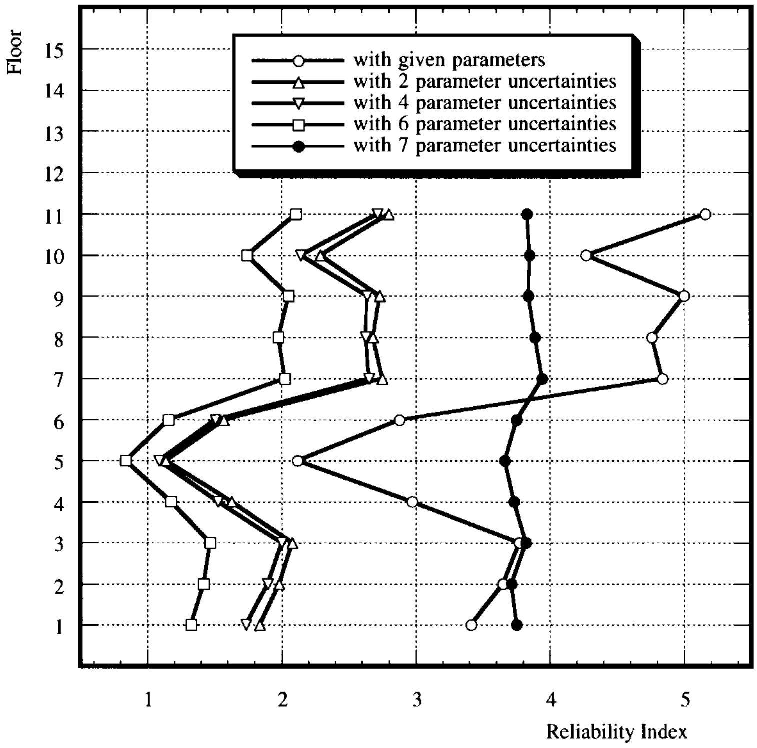

Figure 8. Relationship between floor number and reliability index

damping ratio ξ, the peak acceleration a_g, the duration D and the response spectra S_A, are seen to be very much different than their initial values. The values at the design point corresponding to the random variables that have relatively little uncertainty, i.e. the initial stiffness x_K, the floor weight x_W are, on the other hand, not changed much from their initial values. The influence of the random variables that have relatively larger uncertainty are more dominant than those of the random variables that have relatively little uncertainty. The values corresponding to the peak acceleration a_g, at the design point are seen to be most different from the initial values, meaning a_g has the most dominant effect on the results of reliability evaluation.

The first-order reliability indices that include all of the seven parameter uncertainties are depicted in Figure 8. It can be seen from this figure that the differences in reliabiity indices between different floors are much smaller than in the case when parameters are given. Therefore, the maximum response does not have dominant effect in the case of time-variant reliability analysis as in the case of given parameters. A similar conclusion was obtained for the limit state condition.[42]

For the purpose of comparison, the first-order reliability indices that include two parameter uncertainties (only x_K and x_Q), those that include four parameter uncertainties (x_K, x_Q, x_W and x_ξ) and those that include six parameter uncertainties (x_K, x_Q, x_W,x_ξ, x_D and x_S) under the same level of earthquake input (818 gal) are also depicted in Figure 8. The figure shows that the greater the number of parameter uncertainties included, the smaller the first-order reliability index. In other words, disregarding parameter uncertainties will result in a very high evaluation of the safety of structures. The reliability index with the uncertainties of all the seven parameters are much larger than those with uncertainties of two, four, six parameters, because the latter take very large peak ground acceleration (818 gal) as input.

6. CONCLUSIONS

The response uncertainty evaluation in the present study reveals the following:

(1) An evaluation procedure for the uncertainty of the non-linear stochastic response was developed, in which the mean value and standard deviation of the maximum response that takes into account parameter uncertainties can be evaluated through only a few repetitions of non-linear random vibration analysis using deterministic parameters, without the need for sensitivity analysis of the maximum response.
(2) The response uncertainties evaluated using point estimates are in good agreement with those obtained using MCS. Both the approximation functions for point estimate can be used to evaluate the stochastic response uncertainty with consideration of parameter uncertainties.
(3) The mean values of the maximum response obtained using parameter uncertainties remain almost unchanged as those obtained using deterministic parameters. For the standard deviation of the maximum response, the deviations obtained using parameter uncertainties are very different from those obtained using deterministic parameters.
(4) The effects of the uncertainties contained in the excitation characteristics are greater than those contained in the structural properties.
(5) The greater the number of parameter uncertainties included, the larger the standard deviation of the maximum response. Disregarding parameter uncertainties will evaluate the uncertainties of the maximum response to be very low.
(6) Mean value and standard deviation obtained using point estimates are in very good agreement with the exact values for functions of random variables with small deviation, but for third- and fourth-order moments, the results obtained using point estimates cannot used as an approximation of the actual values. This precaution is necessary when using point estimates.

The time-variant reliability analysis in the present study reveals for the following:

(1) A time-variant reliability analysis procedure for hysteretic MDF structures was developed, in which the failure probability takes into account parameter uncertainties, including random variables that have no CDF. The procedure can be carried out through a several repetitions of earthquake reliability analysis with given parameters, and without any sensitivity analysis.
(2) Because the response surface function is expressed directly as a second-order polynomial of standard normal random variables, the computational accuracy can be improved easily by using SORM, The complicated computation of the Hessian matrix is not required, nor is it necessary to carry out the rotational transformation or eigenvalue analysis of the Hessian matrix.

(3) The failure probability that takes parameter uncertainties into account is much larger than that with deterministic parameters (mean values of random variables). Good agreement was observed between the results obtained using the developed procedure and those obtained using MCS.
(4) The difference between the reliability index with given parameters and that with parameter uncertainties increases as the reliability index increases.
(5) The maximum response has more significant effect on the failure probability when using given parameters than when parameter uncertainties are taken into account.
(6) The influence in the time-variant reliability analysis of the random variables that have relatively larger uncertainty is more dominant than that of random variables that have relatively little uncertainty. The peak ground acceleration has the most dominant effect on the results of reliability evaluation.
(7) The greater the number of parameter uncertainties included, the smaller the first-order reliability index. Disregarding parameter uncertainties will result in very high evaluation of the safety of structures.
(8) The developed procedure in the present paper is generally practical for time-variant reliability analysis of hysteretic MDF structures in which random variables that have no CDF are included. However, like EFORM, the procedure is difficult to apply to problems in which the conditional failure probability $P_f(\mathbf{X})$ is too small.

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