System Reliability Assessment by Method of Moments

Yan-Gang Zhao, M.ASCE,[1] and Alfredo H-S. Ang Hon.M.ASCE[2]

Abstract: The computational assessment of system reliability of structures has remained a challenge in the field of reliability engineering. Calculation of the failure probability for a system is generally difficult even if the potential failure modes are known or can be identified, because available analytical methods require determination of the sensitivity of performance functions, information on mutual correlations among potential failure modes, and determination of design points. In the present paper, a method based on moment approximations is proposed for structural system reliability assessment that is applicable to both series and nonseries systems. The point estimate method is applied to evaluate the first few moments of the system performance function of a structure from which the moment-based reliability index and failure probability can be evaluated without Monte Carlo simulations. The procedure does not require the computation of derivatives, nor determination of the design point and computation of mutual correlations among failure modes; thus, it should be computationally effective for structural assessment of system reliability.

DOI: 10.1061/(ASCE)0733-9445(2003)129:10(1341)

CE Database subject headings: Structural reliability; Failure modes; Moments; Assessment.

Introduction

The evaluation of system reliability for structures has been an active area of research for over three decades. The calculation of the failure probability for a system is generally difficult even if the potential failure modes are known or can be identified, because of the large number of potential failure modes for most practical structures, the difficulty in obtaining the sensitivity of the performance function, and the mutual correlations among failure modes. The search for efficient computational procedures for estimating system reliability has resulted in several approaches such as bounding techniques, the probabilistic network evaluation technique (PNET), and direct or smart Monte Carlo simulations. In the present paper, a computationally more effective method using moment approximations for system reliability is proposed and examined for both series and nonseries systems.

Assessment of System Reliability

A structural system will invariably have multiple modes of potential failure, e.g., E_1, $E_2, \ldots, E_m$. The occurrence of one or more of these failure modes will constitute failure of the system, i.e., system failure is the union of all the modes or $E_1 U E_2 U, \ldots, U E_m$.

For a structural system, each of the failure modes, E_i, can be defined by a performance function $g_i = g_i(\mathbf{X})$ such that $E_i = (g_i < 0)$ and the failure probability of the system is then

$$P_F = \text{Prob}[g_1 \leqslant 0 \cup g_2 \leqslant 0 \cup, \ldots, \cup g_m \leqslant 0] \tag{1}$$

Conversely, the safety of a system is the event in which none of the m potential failure modes occurs; again in the case of a series system, this means

$$\begin{aligned} P_S &= \text{Prob}[g_1 > 0 \cap g_2 > 0 \cap, \ldots, \cap g_m > 0] \\ &= \text{Prob}[\min(g_1, g_2, \ldots, g_m) > 0] \end{aligned} \tag{2}$$

Thus the performance function of a series system, G, can be expressed as the minimum of the performance functions that corresponds to all the potential failure modes, that is,

$$G(\mathbf{X}) = \min[g_1, g_2, \ldots, g_m] \tag{3}$$

where $g_i = g_i(\mathbf{X})$ is the performance function of the ith failure mode.

In the case of a series system, the performance functions of the individual failure modes will be smooth; for a nonseries system, however, each of the failure modes will generally involve combinations of the maximum and minimum of the component performance functions, as illustrated later in example 4. Consequently, the resulting system performance function, $G(\mathbf{X})$, will not be smooth and will be more complex than that of a comparable series system.

Since it is difficult to obtain the sensitivity of the performance function even for a series system like Eq. (3), derivative-based FORM would not be applicable. The failure probability of a system can be determined using bounding techniques (see, e.g., Cornell 1966) as a function of the failure probability of individual modes; however, for a complex system the bounds would be wide even though these bounds can be improved by second-order bounds (Ditlevsen 1979). The failure probability of a system may also be estimated approximately with the probabilistic network evaluation technique developed by Ang and Ma (1981), in which mutual correlations among the failure modes have to be computed. Other methods have been reviewed or discussed, e.g., by Moses (1982), Thoft-Christensen and Murotsu (1986), and Bennett and Ang (1986).

In the present paper, moment approximations (Zhao and Ono

[1]Associate Professor, Dept. of Architecture, Nagoya Institute of Technology, Gokiso-cho, Shyowa-ku, Nagoya 466-8555, Japan.

[2]Research Professor, Dept. of Civil and Environmental Engineering, Univ. of California, Irvine, CA 92697-2175.

Note. Associate Editor: Jamshid Mohammadi. Discussion open until March 1, 2004. Separate discussions must be submitted for individual papers. To extend the closing date by one month, a written request must be filed with the ASCE Managing Editor. The manuscript for this paper was submitted for review and possible publication on May 1, 2002; approved on November 6, 2002. This paper is part of the ***Journal of Structural Engineering***, Vol. 129, No. 10, October 1, 2003. ©ASCE, ISSN 0733-9445/2003/10-1341–1349/$18.00.

2001) for evaluation of system reliability are investigated. If the central moments of the system performance function, described in Eq. (3) for a series system, can be obtained, the failure probability of a system, which is defined as Prob$[G(\mathbf{X})<0]$ can be expressed as a function of the central moments. The proposed method, therefore, is based on the premise that by finding the relationship between the failure probability and the central moments of $G(\mathbf{X})$, the probability of system failure can be assessed. Because the first two moments are generally inadequate, high-order moments will invariably be necessary.

Approximating the distribution of a random variable using its moments of finite order is a well known problem in statistics, and various approximations such as the Pearson, Johnson, and Burr systems and the Edgeworth and Cornish–Fisher expansions were developed (Stuart and Ord 1987). Their application in structural reliability has been examined by Winterstein (1988) and by Parkinson (1978). A first-order third-moment reliability method (Tichy 1994) was developed, the results of which, however, depend on a successful search for the design point. A method of moments that is independent of the design point was developed by Grigoriu and Lind (1980) that requires optimal estimation of convolution integrals using higher-order moments of the performance function and a linear combination of distributions from prescribed reference sets. Grigoriu (1983) also developed a procedure to estimate the failure probability using a method of moments, in which moments were obtained from Monte Carlo simulations. Hong (1996) proposed a point-estimate moment-based reliability analysis method, in which the concentrations in the point estimates require the solution of nonlinear equations.

In the present paper, the moments of the system performance function, $G(\mathbf{X})$, are obtained using point estimates in standard normal space in which the concentrations can be readily obtained without solving nonlinear equations. After the moments of the performance function $G(\mathbf{X})$ are obtained, the moment-based reliability index can then be evaluated using an available standardization function or appropriate distribution systems.

Determining Moments of Performance Function

In the present paper, the point-estimate method (Zhao and Ono 2000a) is used to determine the moments of the system performance function like in Eq. (3) for a series system. For a function of only one random variable $y=y(x)$, the moments of y can be point estimated as

$$\mu_y=\sum_{k=1}^{m} P_k y[T^{-1}(u_k)] \tag{4a}$$

$$\sigma_y^2=\sum_{k=1}^{m} P_k\{y[T^{-1}(u_k)]-\mu_y\}^2 \tag{4b}$$

$$\alpha_{ry}\sigma_y^r=\sum_{k=1}^{m} P_k\{y[T^{-1}(u_k)]-\mu_y\}^r \tag{4c}$$

where μ_y, σ_y, and α_{ry} = mean value, standard deviation, and rth dimensionless central moment of $y(x)$, T^{-1} = inverse Rosenblatt transformation. u_1, $u_2,\ldots,u_m$ = estimating points and P_1, $P_2,\ldots,P_m$ = corresponding weights.

The estimating points u_i and their corresponding weights P_i can be readily obtained as (Zhao and Ono 2000a)

$$u_i=\sqrt{2}x_i \quad P_i=\frac{w_i}{\sqrt{\pi}} \tag{5}$$

where x_i and w_i are the abscissas and weights for Hermite integration with weight function $\exp(-x^2)$ that can be found in work by Abramowitz and Stegum (1972).

For a five point estimate in standard normal space (Zhao and Ono 2000a),

$$u_0=0 \quad P_0=8/15 \tag{6a}$$

$$u_{1+}=-u_{1-}=1.355\,6262 \quad P_1=0.222\,0759 \tag{6b}$$

$$u_{2+}=-u_{2-}=2.856\,9700 \quad P_2=1.125\,74\times10^{-2} \tag{6c}$$

whereas for a seven point estimate in standard normal space,

$$u_0=0 \quad P_0=16/35 \tag{7a}$$

$$u_{1+}=-u_{1-}=1.154\,4054 \quad P_1=0.240\,1233 \tag{7b}$$

$$u_{2+}=-u_{2-}=2.366\,7594 \quad P_2=3.075\,71\times10^{-2} \tag{7c}$$

$$u_{3+}=-u_{3-}=3.750\,4397 \quad P_3=5.482\,69\times10^{-4} \tag{7d}$$

For a function of many variables $Z=G(\mathbf{X})$, where $\mathbf{X}=x_1,x_2,\ldots,x_n$, the joint probability density is assumed to be concentrated at points in the m^n hyperquadrants of the space defined by the n random variables, in which m is the number of estimating points used in the point estimates for functions of respective single random variables. Then the moments of $Z=G(\mathbf{X})$ can be point estimated as

$$\mu_G=\sum\prod_{i=1}^{n} P_{ci}G[T^{-1}(u_{c1},u_{c2},\ldots,u_{cn})] \tag{8a}$$

$$\sigma_G^2=\sum\prod_{i=1}^{n} P_{ci}[G(T^{-1}(u_{c1},u_{c2},\ldots,u_{cn}))-\mu_G]^2 \tag{8b}$$

$$\alpha_{rG}\sigma_G^r=\sum\prod_{i=1}^{n} P_{ci}[G(T^{-1}(u_{c1},u_{c2},\ldots,u_{cn}))-\mu_G]^r \tag{8c}$$

where c = distinct combination of n terms from a group $[1,2,\ldots,m]$ and $ci=i$th term of c. $u_{ci}=ci$th estimating point and P_{ci} = weight corresponding to u_{ci}. n=number of random variables and m=number of estimating points, where μ_G, σ_G, and α_{rG} are the mean value, standard deviation, and the rth dimensionless central moment of $G(\mathbf{X})$, and T^{-1} is the inverse Rosenblatt transformation.

Since all distinct combinations have to be considered, m^n times of function calls for computing $G(\mathbf{X})$ are required. The computations involved in Eqs. (8), therefore, can be massive when n is large. In order to avoid this problem, the function $G(\mathbf{X})$ may be approximated by $G^*(\mathbf{X})$ as follows (Zhao and Ono 2000a):

$$G^*(\mathbf{X})=\sum_{i=1}^{n}(G_i-G_\mu)+G_\mu \tag{9a}$$

where

$$G_\mu=G(\boldsymbol{\mu}) \tag{9b}$$

$$G_i=G[T^{-1}(\mathbf{U}_i)] \tag{9c}$$

where $\boldsymbol{\mu}$ represents the vector in which all the random variables take their mean values, and $\mathbf{U}_i=[u_{\mu1},u_{\mu2},\ldots,u_{\mu i-1},u_i,u_{\mu i+1},\ldots,u_{\mu n}]^T$, where $u_{\mu k}$, $k=1,\ldots,n$ except i, is the kth value of u_μ, which is the vector in u space that corresponds to $\boldsymbol{\mu}$. G_μ is a constant and G_i is a function of only u_i

for specific $G^*(\mathbf{X})$. T^{-1} is the inverse Rosenblatt transformation. For independent random variables $\mathbf{X}$, G_i can simply be expressed as

$$G_i = G(\mu_1, \mu_2, \ldots, \mu_{i-1}, x_i, \mu_{i+1}, \ldots, \mu_n) \tag{9d}$$

The approximation of Eq. (9*a*) can be viewed as a generalization of the following observation: If $G(\mathbf{X})$ is of the form

$$G(\mathbf{X}) = \sum_{i=1}^{n} a_i x_i \quad \text{or} \quad G(\mathbf{X}) = \sum_{i=1}^{n} y_i(x_i) \tag{9e}$$

where a_i = constant; and y_i = arbitrary function of x_i, Eq. (9*a*) will become exact, i.e., $G^*(X) = G(X)$.

Observe that u_i and $i = 1,...,n$ are independent and G_i is a function only of u_i; therefore, G_i, $i = 1,...,n$ are also independent. Hence, the first four moments of $G^*(\mathbf{X})$ in Eq. (9*a*) can be expressed as

$$\mu_G = \sum_{i=1}^{n} (\mu_i - G_\mu) + G_\mu \tag{10a}$$

$$\sigma_G^2 = \sum_{i=1}^{n} \sigma_i^2 \tag{10b}$$

$$\alpha_{3G}\sigma_G^3 = \sum_{i=1}^{n} \alpha_{3i}\sigma_i^3 \tag{10c}$$

$$\alpha_{4G}\sigma_G^4 = \sum_{i=1}^{n} \alpha_{4i}\sigma_i^4 + 6\sum_{i=1}^{n-1}\sum_{j>i}^{n} \sigma_i^2\sigma_j^2 \tag{10d}$$

where μ_i and σ_i = mean value and standard deviation of G_i, respectively. α_{3i} and α_{4i} are the third and fourth dimensionless central moments, i.e., the skewness and kurtosis of G_i.

Since G_i is a function of only one standard normal random variable u_i, the first four moments, μ_i, σ_i, α_{3i}, and α_{4i}, can be point estimated from Eqs. (4). For a performance function $G(\mathbf{X})$ with n variables, if the probability moments of G_i are estimated using an m-point estimate, only mn function calls of $G(\mathbf{X})$ are required to estimate the first three or four moments of $G(\mathbf{X})$. After the first three or four moments of $G(\mathbf{X})$ are obtained, the reliability analysis becomes a problem of approximating the distribution of a specific random variable with its known first three or four moments.

Moment-Method Formulas

Third-Moment Reliability Index

The third-moment reliability index can be obtained from some three-parameter distributions, such as the three-parameter log–normal distribution (Tichy 1994) and the third-moment transformation (Zhao and Ono 2000b). These distributions give similar results when the absolute skewness of the performance function is small (e.g., $\alpha_3 < 1$). In the present paper, the three-parameter log–normal distribution is used. For the performance function $z = G(\mathbf{X})$ described in Eq. (3), if the first three moments are obtained, and assuming that the standardized variable

$$z_u = \frac{z - \mu_G}{\sigma_G} \tag{11}$$

obeys the three-parameter lognormal distribution (Tichy 1994), the standard normal random variable u is expressed as the following function:

$$u = \frac{\operatorname{sign}(\alpha_{3G})}{\sqrt{\ln(A)}} \ln\left[\sqrt{A}\left(1 - \frac{z_u}{u_b}\right)\right] \tag{12}$$

where μ_G and σ_G = mean and standard deviations of z, respectively; and u_b = standardized bound of the distribution.

$$A = 1 + \frac{1}{u_b^2} \tag{13}$$

The relationship between the bound u_b and the skewness α_{3G} is given by

$$\alpha_{3G} = -\left(3 + \frac{1}{u_b^2}\right)\frac{1}{u_b} \tag{14}$$

The solution of Eq. (14) yields

$$u_b = (a+b)^{1/3} + (a-b)^{1/3} - \frac{1}{\alpha_{3G}} \tag{15a}$$

in which

$$a = -\frac{1}{\alpha_{3G}}\left(\frac{1}{\alpha_{3G}^2} + \frac{1}{2}\right) \quad b = -\frac{1}{2\alpha_{3G}^2}\sqrt{\alpha_{3G}^2 + 4} \tag{15b}$$

Since

$$\text{Prob}[z \leqslant 0] = \text{Prob}[z_u \leqslant -\beta_{2M}] \tag{16}$$

the third-moment reliability index (3M reliability index) is (Zhao and Ono 2001)

$$\beta_{3M} = \frac{-\operatorname{sign}(\alpha_{3G})}{\sqrt{\ln(A)}} \ln\left[\sqrt{A}\left(1 + \frac{\beta_{2M}}{u_b}\right)\right] \tag{17}$$

where β_{2M} = second-moment reliability index (2M reliability index).

Simplification of the 3M Reliability Index

According to Eq. (12), the absolute value of the random variable $(z_u - u_b)$ obeys the log–normal distribution with parameters λ and ζ, and the coefficient of skewness α_{3G} is given by

$$\alpha_{3G} = [\exp(\zeta^2) + 2]\sqrt{\exp(\zeta^2) - 1} \tag{18}$$

Comparing Eq. (18) with Eq. (14), one can easily see that

$$u_b = -\operatorname{sign}(\alpha_{3G})[\exp(\zeta^2) - 1]^{-1/2} \tag{19}$$

When the absolute value of α_{3G} is less than 1, the absolute value of ζ is less than 0.314, and the following approximation applies within error of less than 2.5%:

$$\sqrt{\exp(\zeta^2) - 1} = \zeta \tag{20}$$

Substituting Eq. (20) into Eq. (18), one obtains

$$\alpha_{3G} = 3\zeta + \zeta^3 \tag{21}$$

For small ζ (e.g., $\zeta < 0.314$), Eq. (21) yields the following approximations for ζ and u_b:

$$u_b = -\frac{3}{\alpha_{3G}} \quad \zeta = 1/3\alpha_{3G} \tag{22}$$

Then, the standard normal random variable u can be expressed as follows:

$$u = \frac{\alpha_{3G}}{6} + \frac{3}{\alpha_{3G}} \ln\left[1 + \frac{1}{3}\alpha_{3G} z_u\right] \tag{23}$$

and the 3M reliability index becomes

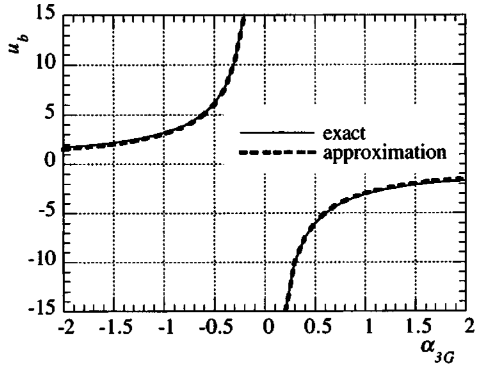

Fig. 1. Variations of standardized bound

$$\beta_{3M} = -\frac{\alpha_{3G}}{6} - \frac{3}{\alpha_{3G}} \ln\left[1 - \frac{1}{3}\alpha_{3G}\beta_{2M}\right] \tag{24a}$$

Observe that as x approaches 0, $\ln(1+x)=x$, and Eq. (24*a*) becomes

$$\beta_{3M} = \beta_{2M} - 1/6\alpha_{3G} \tag{24b}$$

This implies that β_{3M} approaches β_{2M} for extremely small α_{3G}.

For negative α_{3G}, Eq. (24*a*) is valid for any values of β_{2M}. However, for positive α_{3G}, Eq. (24*a*) is valid only if $\beta_{2M} < 3/\alpha_{3G}$.

To examine the accuracy of the approximation expressed in Eq. (22), the values of the standardized bound u_b are depicted in Fig. 1, where the thin solid lines indicate the exact values of u_b obtained from Eq. (15) and the thick dash lines indicate those obtained with Eq. (22). From Fig. 1, one can see that although Eq. (22) is much simpler than Eq. (15), the u_b results obtained with Eq. (22) agree well with those obtained from Eq. (15).

The accuracy of the approximate $3M$ reliability index in Eqs. (24) is also demonstrated in Fig. 2, where the thin solid lines indicate the exact reliability indices obtained with Eq. (17) and the dashed lines indicate those obtained with Eqs. (24). From Fig. 2, one can see that the reliability indices obtained with Eqs. (24) also agree well with those obtained from Eq. (17). One can also see that the $2M$ reliability index (shown as dash-dotted lines in Fig. 2) contains significant error.

Fourth-Moment Reliability Index

The fourth-moment reliability index can be obtained utilizing existing systems of frequency curves, such as the Pearson, Johnson, and Burr systems (Stuart and Ord 1987; Hong 1996), and Ramberg's lamda distribution (Grigoriu 1983). Since the quality of approximating the tail area of a distribution is relatively insensitive to the distribution family selected (Pearson et al. 1979) and the solution of nonlinear equations is necessary to determine the parameters of the Johnson and Burr systems or the lamda distribution, the Pearson system is selected for use in the present study. For the standardized variable z_u of Eq. (11), the probability density function (PDF) of z_u and f, satisfies the following differential equation in the Pearson system (Stuart and Ord 1987):

$$\frac{1}{f}\frac{df}{dz_u} = -\frac{az_u + b}{c + bz_u + dz_u^2} \tag{25}$$

where

$$a = 10\alpha_{4G} - 12\alpha_{3G}^2 - 18$$

$$b = \alpha_{3G}(\alpha_{4G} + 3)$$

$$c = 4\alpha_{4G} - 3\alpha_{3G}^2$$

$$d = 2\alpha_{4G} - 3\alpha_{3G}^2 - 6$$

Using the relationship described in Eq. (16), the fourth-moment reliability index ($4M$ reliability index) is given by

$$\beta_{4M} = -\Phi^{-1}\left[\int_{-\infty}^{-\beta_{2M}} f(z_u)\,dz_u\right] \tag{26}$$

The PDF of z_u, depending on the values of parameters a, b, c, and d (Zhao and Ono 2001) is as follows:

$$f(z_u) = K(z_u - r_2)^{-1/\sqrt{\Delta}(ar_2+b)}(r_1 - z_u)^{1/\sqrt{\Delta}(ar_1+b)}$$

$$\text{for } \Delta > 0,\ d < 0 \tag{27a}$$

$$f(z_u) = K(c + bz_u)^{(ac-b^2)/b^2}\exp\left[-\frac{az_u}{b}\right] \quad \text{for } \Delta > 0,\ d = 0 \tag{27b}$$

$$f(z_u) = K|z_u - r_1|^{1/\sqrt{\Delta}(ar_1+b)}|z_u - r_2|^{-1/\sqrt{\Delta}(ar_2+b)}$$

$$\text{for } \Delta > 0,\ d > 0 \tag{27c}$$

$$f(z_u) = K|z_u - r_0|^{-a/d}\exp\left[\frac{ar_0 + b}{d(z_u - r_0)}\right] \quad \text{for } \Delta = 0 \tag{27d}$$

$$f(z_u) = K(c + bz_u + dz_u^2)^{-a/2d}\exp\left[\frac{ab - 2bd}{d\sqrt{-\Delta}}\tan^{-1}\left(\frac{b + 2dz_u}{\sqrt{-\Delta}}\right)\right]$$

$$\text{for } \Delta < 0 \tag{27e}$$

where K is determined from $F(+\infty) = 1$, and

$$\Delta = b^2 - 4cd \quad r_1 = \frac{-b - \sqrt{\Delta}}{2d} \quad r_2 = \frac{-b + \sqrt{\Delta}}{2d} \quad r_0 = \frac{-b}{2d} \tag{27f}$$

One can note that when $\alpha_{3G} = 0$ and $\alpha_{4G} = 3$, z_u becomes a standard normal variable; in this case, $\beta_{4M} = \beta_{2M}$.

Numerical Examples

A number of examples are illustrated below to demonstrate the accuracy and computational effectiveness of the moment-based method. These include problems that involve normal as well as non-normal probability distributions. If one or more of the random variables are non-normal, appropriate Rosenblatt transforma-

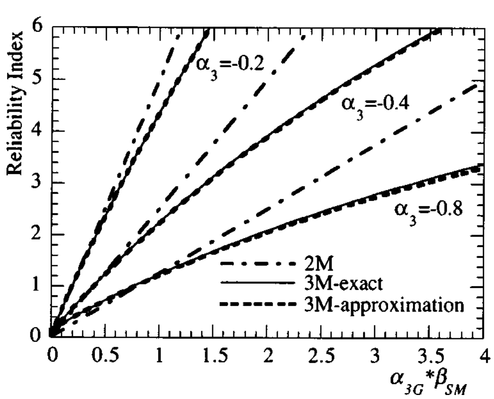

Fig. 2. Variations of $3M$ reliability index

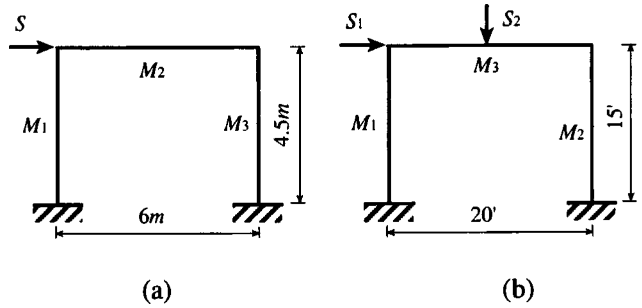

Fig. 3. One-story one-bay frames of examples 1 and 2

tions are necessary (Ang and Tang 1984) to evaluate the moments as indicated in Eqs. (4) and (8). Most of the examples are series systems with elastoplastic components, however, two nonseries systems are illustrated in example 4 and they involve brittle components. One of the case in example 2 also illustrates the limitation of the first four moments for problems that involve extremely small failure probability.

Example 1

In the first example we consider a one-story one-bay elastoplastic frame, shown in Fig. 3(a). This series system is used to illustrate numerical details of the procedure for the proposed moment method. The four potential failure modes of the system can be readily identified and defined by four linear performance functions. The FORM reliability indices for the respective failure modes are given in parentheses below to indicate the relative dominance of the four different modes.

$$g_1(\mathbf{X})=2M_1+2M_3-4.5S \quad (\beta_F=3.334) \tag{28a}$$

$$g_2(\mathbf{X})=2M_1+M_2+M_3-4.5S \quad (\beta_F=3.364) \tag{28b}$$

$$g_3(\mathbf{X})=M_1+M_2+2M_3-4.5S \quad (\beta_F=3.364) \tag{28c}$$

$$g_4(\mathbf{X})=M_1+2M_2+M_3-4.5S \quad (\beta_F=3.364) \tag{28d}$$

Since this is a series system, the performance function of the system can be defined as the minimum of the above, i.e.,

$$G(\mathbf{X})=\min\{g_1(\mathbf{X}),g_2(\mathbf{X}),g_3(\mathbf{X}),g_4(\mathbf{X})\} \tag{28e}$$

where M_i and S = independent log–normal random variables with mean and standard deviations of $\mu_M=200tm$, $\mu_S=50t$, $\sigma_M=30tm$, and $\sigma_S=20t$. Using the mean values of all the variables in Eq. (28*e*), G_μ as defined in Eqs. (9) is

$$G_\mu=\min\{2*200+2*200-4.5*50,2*200+200+200$$
$$-4.5*50,200+200+2*200-4.5*40,200+2*200+200$$
$$-4.5*40\}=\min\{575,575,575,575\}=575$$

Substituting the mean values of all the variables except M_1 into Eq. (28*e*), G_1 as defined in Eqs. (9) becomes

$$G_1=\min\{2M_1+2*200-4.5*50,2M_1+200+200-4.5*50,M_1$$
$$+200+2*200-4.5*40,M_1+2*200+200-4.5*40\}$$
$$=\min\{2M_1+175,2M_1+175,\ M_1+375,M_1+375\}$$
$$=\min\{2M_1+175,\ M_1+375\}$$

Similarly, G_i, $i=2$, 3, 4, are, respectively,

$$G_2=\min\{575,2M_2+175,M_2+375\}$$

$$G_3=\min\{2M_3+175,\ M_3+375\}$$

$$G_4=800-4.5S$$

Since G_1 is a function of single random variable M_1, its moments can be point estimated using Eqs. (4). For a five-point estimate, using the inverse Rosenblatt transformation expressed as

$$T^{-1}(u_i)=F^{-1}[\Phi(u_i)] \tag{29}$$

where F = cumulative distribution function of M_1; and Φ = standard normal probability, the five estimating points of Eqs. (6) can be easily transformed into original space as $T^{-1}(u_{2-})=129.157$, $T^{-1}(u_{1-})=161.576$, $T^{-1}(u_0)=197.787$, $T^{-1}(u_{1+})=242.113$, and $T^{-1}(u_{2+})=302.886$. Substituting these into Eqs. (4) using the corresponding weights listed in Eqs. (6), the first four moments of G_1 are approximately

$$\mu_1=564.489tm,\ \ \sigma_1=44.164tm,\ \ \alpha_{31}=-0.479,\ \ \alpha_{41}=2.943,$$

Similarly the first four moments of G_2, G_3, G_4 are

$$\mu_2=553.979tm,\ \ \sigma_2=33.259tm,\ \ \alpha_{32}=-1.463,\ \ \alpha_{42}=3.785,$$

$$\mu_3=564.489tm,\ \ \sigma_3=44.164tm,\alpha_{33}=-0.479,\ \ \alpha_{43}=2.943,$$

$$\mu_4=575.000tm,\ \ \sigma_4=90.000tm,\alpha_{34}=-1.264,\ \ \alpha_{44}=5.968$$

Then using Eqs. (10), the first four moments of G^* are approximately $\mu_G=532.958$, $\sigma_G=114.485$, $\alpha_{3G}=-0.704$, and $\alpha_{4G}=4.098$.

Finally, with the first two moments of the performance function G^*, the $2M$ reliability index is $\beta_{2M}=4.655$. With the first three moments of the performance function, Eqs. (24) give the $3M$ reliability index as $\beta_{3M}=3.264$ with corresponding failure probability of $P_F=5.498\times10^{-4}$. With the first four moments of the performance function, parameters *a*, *b*, *c*, and *d* defined in Eq. (25) are readily obtained to be $a=17.030$, $b=-4.996$, $c=14.904$, and $d=0.709$. Since $\Delta=b^2-4cd=-17.288<0$, the PDF of the standardized performance function z_u is in the form of Eq. (27*e*). Using $F(+\infty)=1$, K in Eq. (27*e*) is $K=4263.42$ and the PDF of z_u becomes

$$f(z_u)=4263.42(14.904+4.996z_u+0.709z_u^2)^{-12.015}$$
$$\times\exp[-26.472\tan^{-1}(0.341z_u-1.204)] \tag{30}$$

By substituting Eq. (30) and $\beta_{2M}=4.655$ into Eq. (26), the $4M$ reliability index is $\beta_{4M}=3.243$ with corresponding probability of failure of $P_F=5.91\times10^{-4}$.

Using Monte Carlo simulations (MCS) with 1 million samples, the probability of failure for this system is estimated to be 5.34×10^{-4} with a corresponding reliability index of $\beta=3.272$. The coefficient of variation (COV) of this MCS estimate is 4.32%. In this case, only 20 function calls are used (with a total of 20 estimating points for all the variables). For this example, one can see that both the results of the third- and fourth-moment approximations are in close agreement with the MCS results, whereas the second-moment approximation has gross error (42% overestimation of the reliability index).

Example 2

The second example is also a one-story and one-bay elastoplastic frame as shown in Fig. 3(b) (after work by Ono et al. 1990). The statistics of the member strengths and loads are as follows: mean values are $\mu_{M1}=\mu_{M2}=500$ ft kip, $\mu_{M3}=667$ ft kip, μ_{S1}

$=50$ kip, and $\mu_{S2}=100$ kip; standard deviations are $\sigma_{M1}=\sigma_{M2}=75$ ft kip, $\sigma_{M3}=100$ ft kip, $\sigma_{S1}=15$ kip, and $\sigma_{S2}=10$ kip. The performance functions that correspond to the five most likely failure modes obtained from stochastic limit analysis are listed as follows with the FORM reliability index for each mode listed in parentheses to show the relative dominance of the different modes:

$$g_1=M_1+3M_2+2M_3-15S_1-10S_2 \quad (\beta_F=3.551) \tag{31a}$$

$$g_2=2M_1+2M_2-15S_1 \quad (\beta_F=3.247) \tag{31b}$$

$$g_3=M_1+M_2+4M_3-15S_1-10S_2 \quad (\beta_F=3.848) \tag{31c}$$

$$g_4=2M_1+M_2+M_3-15S_1 \quad (\beta_F=3.562) \tag{31d}$$

$$g_5=M_1+M_2+2M_3-15S_1 \quad (\beta_F=3.784) \tag{31e}$$

$$g_6=M_1+2M_2+M_3-15S_1 \quad (\beta_F=3.562) \tag{31f}$$

For this example, the corresponding G_i of Eqs. (9) is as follows:

$$G_\mu=1250 \text{ ft kip}$$

$$G_1=\min\{917+M_1,250+2M_1\}$$

$$G_2=\min\{917+M_2,250+2M_2,84+3M_2\}$$

$$G_3=\min\{1250,750+M_3,250+2M_3,-750+4M_3\}$$

$$G_4=2000-15S_1$$

$$G_5=\min\{1250,2584-10S_2\}$$

Using the point estimate method with five estimating points, the first four moments of G_1, G_2, G_3, G_4, and G_5 are approximately

$$\mu_1=1{,}248.98 \text{ ft kip}, \quad \sigma_1=146.79 \text{ ft kip},$$

$$\alpha_{31}=-0.293, \quad \alpha_{41}=2.767,$$

$$\mu_2=1{,}248.98 \text{ ft kip}, \quad \sigma_2=146.79 \text{ ft kip},$$

$$\alpha_{32}=0.291, \quad \alpha_{42}=2.767,$$

$$\mu_3=1{,}246.88 \text{ ft kip}, \quad \sigma_3=29.229 \text{ ft kip},$$

$$\alpha_{33}=-9.265, \quad \alpha_{43}=88.842,$$

$$\mu_4=1{,}250.00 \text{ ft kip}, \quad \sigma_4=225.00 \text{ ft kip},$$

$$\alpha_{34}=-0.927, \quad \alpha_{44}=4.547,$$

$$\mu_5=1{,}250.00 \text{ ft kip}, \quad \sigma_5=0.00 \text{ ft kip}, \quad \alpha_{35}=\text{--},\alpha_{45}=\text{--},$$

In this case, $\sigma_5=0$, whereas α_{35} and α_{45} cannot be obtained according to Eqs. (4). This is because G_5 is almost a constant and it has almost no influence on the results of σ_G, α_{3G}, and α_{4G}. Any values of α_{35} and α_{45} can be used; e.g., $\alpha_{35}=0$ and $\alpha_{45}=3$, and then use Eqs. (10) as usual, or substitute G_5 as a constant in Eqs. (9). The results would remain the same.

Then using Eqs. (10), the first four moments of G^* are approximately $\mu_G=1{,}244.85$, $\sigma_G=307.523$, $\alpha_{3G}=-0.307$, and $\alpha_{4G}=3.426$.

With these first four moments of the performance function G^*, the $2M$ reliability index is $\beta_{2M}=4.048$, whereas the $3M$ reliability index of Eqs. (24) is $\beta_{3M}=3.437$ with corresponding failure probability of $P_F=2.937\times10^{-4}$. The $4M$ reliability index of Eq. (26) gives $\beta_{4M}=3.276$ with corresponding failure probability of $P_F=5.268\times10^{-4}$. MCS with 1 million samples gives a probability of failure for this system of 6.45×10^{-4} with corresponding reliability index of $\beta=3.218$. The COV of this MCS estimate is 3.94%. For this example, the $3M$ reliability index errs about 6%, whereas the results of the fourth-moment approximation is in close agreement with the MCS results. Again the $2M$ reliability index has a significant error of about 25%.

Table 1. Computational Results for Example 2 with Different Types of PDFs

Variable	Weibull type	Gamma type	Gumbel type	Normal type
μ_G	1,242.12	1,244.58	1,244.41	1,243.58
σ_G	317.852	308.823	302.810	311.894
α_{3G}	−0.216	−0.207	−0.310	−0.054
α_{4G}	3.199	3.138	3.765	3.038
β_{2M}	3.908 (16%)	4.030 (15%)	4.110 (30%)	3.987 (5%)
β_{3M}	3.480 (3%)	3.590 (2%)	3.476 (10%)	3.861 (2%)
P_F	2.508×10^{-4}	1.652×10^{-4}	2.543×10^{-4}	5.651×10^{-5}
β_{4M}	3.381 (0%)	3.528 (0%)	3.178 (1%)	3.863 (2%)
P_F	3.614×10^{-4}	2.094×10^{-4}	7.145×10^{-4}	5.591×10^{-5}
β_{MCS}	3.379	3.515	3.151	3.794
P_F	3.63×10^{-4}	2.2×10^{-4}	8.14×10^{-4}	7.4×10^{-5}
COV of P_F	5.2%	6.7%	3.5%	11.6%

Note: Percentage of error in the reliability index relative to that of MCS is in parenthesis.

The reliability analyses for this example (example 2) were extended and different types of distribution of the random variables were assumed. Assuming all the member strengths and loads are Weibull random variables, the results of the moment method and of the MCS with 1 million samples are summarized in column 2 of Table 1. Results for gamma, Gumbel, and normal distributed random variables are also summarized in columns 3–5, respectively, in Table 1. From Table 1, one can observe that irrespective of the types of distribution, both the $3M$ and $4M$ reliability indices are in close agreement with the MCS results. The $2M$ reliability indices, however, consistently contain significant error.

Finally, this example is extended further to examine the applicability (and limitation) to problems with extremely small probability of failure; to do this, the mean loads are assumed to be $\mu_{S1}=35$ kip and $\mu_{S2}=75$ kip. Using the seven-point estimate, the first four moments of G^* are approximately $\mu_G=1{,}470.13$, $\sigma_G=260.32$, $\alpha_{3G}=-0.096$, and $\alpha_{4G}=3.186$. With these first four moments of the system performance function, the $2M$ reliability index is $\beta_{2M}=5.647$, and the $3M$ reliability index is $\beta_{3M}=5.207$ with corresponding failure probability of $P_F=9.58\times10^{-8}$. The $4M$ reliability index is found to be $\beta_{4M}=4.652$ with corresponding failure probability of $P_F=1.64\times10^{-6}$. Using MCS with 30 million samples, the probability of failure for this system is 5.333×10^{-6} with corresponding reliability index of $\beta=4.403$. The COV of this MCS estimate is 7.91%. For this example, the $3M$ and $4M$ reliability indices contain errors of about 18 and 5.6%, respectively, whereas the $2M$ reliability index overestimates the correct value by about 28%. It is interesting to observe that in this case of very small failure probability, the accuracy of the $4M$ reliability index has deteriorated.

Example 3: Two-Story One-Bay Truss Structure

The third example is an elastoplastic truss structure with two stories and one bay, shown in Fig. 4, which is also a series system. The statistics of the member strengths and loads are as fol-

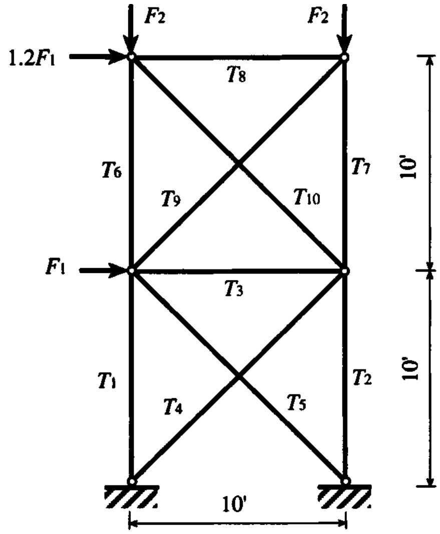

Fig. 4. Two-story one-bay truss

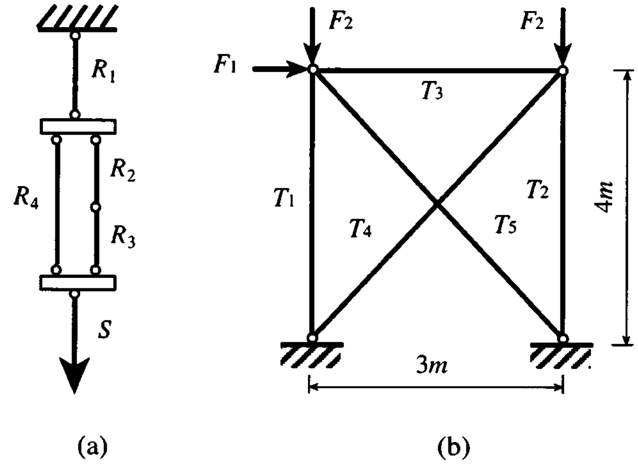

Fig. 5. (a) Brittle chain system and (b) brittle truss

lows: mean values are $\mu_{T1}=\mu_{T2}=90$ kip, $\mu_{T3}=9$ kip, $\mu_{T4}=\mu_{T5}=48$ kip, $\mu_{T6}=\mu_{T7}=21$ kip, $\mu_{T8}=15$ kip, $\mu_{T9}=\mu_{T10}=30$ kip, $\mu_{F1}=11$ kip, and $\mu_{F2}=3.6$ kip; the coefficients of variation are $V_{T1}=\ldots=V_{T10}=0.15$, $V_{F1}=0.3$, and $V_{F2}=0.2$. The performance functions that correspond to the eight most likely failure modes are given below (after work by Ono et al. 1990), with the respective FORM reliability indices listed in parentheses showing that none of the modes are significantly dominant.

$$g_1=0.7071T_4+0.7071T_5-2.2F_1 \quad (\beta_F=3.409) \tag{32a}$$

$$g_2=T_6+0.7071T_{10}-1.2F_1-F_2 \quad (\beta_F=3.497) \tag{32b}$$

$$g_3=T_3+0.7071T_5+0.7071T_{10}-2.2F_1 \quad (\beta_F=3.264) \tag{32c}$$

$$g_4=T_8+0.7071T_{10}-1.2F_1 \quad (\beta_F=3.333) \tag{32d}$$

$$g_5=T_6+T_7-1.2F_1 \quad (\beta_F=3.814) \tag{32e}$$

$$g_6=T_3+0.7071T_5-1.2F_1-F_2 \quad (\beta_F=3.484) \tag{32f}$$

$$g_7=0.7071T_9+0.7071T_{10}-1.2F_1 \quad (\beta_F=3.846) \tag{32g}$$

$$g_8=T_1+0.7071T_5-3.4F_1-F_2 \quad (\beta_F=3.793) \tag{32h}$$

Using the five-point estimate, the first four moments of G^* are approximately $\mu_G=22.316$, $\sigma_G=5.379$, $\alpha_{3G}=-0.408$, and $\alpha_{4G}=3.566$. With these first four moments of the system performance function, the moment-based reliability indices are $\beta_{2M}=4.148$ and $\beta_{3M}=3.356$ with $P_F=3.942\times10^{-4}$ and $\beta_{4M}=3.229$ with $P_F=6.213\times10^{-4}$. Using MCS with 1 million samples, the probability of failure for this system is 7.86×10^{-4} with a corresponding reliability index of $\beta=3.161$. The COV of this MCS estimate is 3.57%. For this example, the $3M$ reliability index errs about 6%, whereas the $4M$ reliability index errs about 2%. The $2M$ reliability index has a significant error of about 30%.

Example 4: Two Brittle Systems

Illustrated next are two brittle systems that are nonseries systems, a simple parallel-chain system and a truss system, shown in Figs. 5(a and b), respectively. Assume that the individual components of each system will fail by tensile fracture or compressive buckling, and thus may be assumed to be brittle, i.e., once failure occurs the strength of a component is reduced to zero. In these two cases, each of the systems is a nonseries system.

For the parallel-chain system shown in Fig. 5(a) there are two failure modes with respective performance functions of

$$g_1=R_1-S \tag{33a}$$

$$g_2=\max\{\min(R_2,R_3,R_4)-1/2S,\max[\min(R_2,R_3),R_4]-S\} \tag{33b}$$

in which fracture strength R_i and load S are independent log–normal random variables with means deviations of $\mu_{R1}=2{,}200$ kg, $\mu_{R2}=2{,}100$ kg, $\mu_{R3}=2{,}300$ kg, $\mu_{R4}=2{,}000$ kg, and $\mu_S=1{,}200$ kg, and standard deviations of $\sigma_{R1}=220$ kg, $\sigma_{R2}=210$ kg, $\sigma_{R3}=230$ kg, $\sigma_{R4}=20$ kg, and $\sigma_S=240$ kg.

Similar to, in Fig. 5(a) for the truss in Fig. 5(b), the corresponding performance functions for each of the failure modes can be shown to be those listed below:

$$g_1=\max[T_1-1.127F_1+0.826F_2,\min(T_2-4/3F_1,T_3-F_1+3/4F_2,T_4-5/4F_2,T_5-5/3F_1+5/4F_2)] \tag{34a}$$

$$g_2=\max[T_2-0.206F_1-0.826F_2,\min(T_1-4/3F_1,T_3-3/4F_2,T_4-5/3F_1-5/4F_2,T_5-5/4F_2)] \tag{34b}$$

$$g_3=\max[T_3-0.155F_1+0.13F_2,\min(T_1-4/3F_1+F_1,T_2-F_2,T_4-5/3F_1)] \tag{34c}$$

$$g_4=\max[T_4-1.409F_1-0.217F_2,\min(T_1-F_2,T_2-4/3F_1-F_2,T_3-F_1,T_5-5/3F_2)] \tag{34d}$$

$$g_5=\max[T_5-0.258F_1+0.217F_2,\min(T_1-4/3F_1+F_2,T_2-F_2,T_4-5/3F_1)] \tag{34e}$$

where fracture strength T_i and load F_i are independent log–normal random variables with means and standard deviations of $\mu_{T1}=\mu_{T2}=40t$, $\mu_{T3}=10t$, $\mu_{T4}=\mu_{T5}=20t$, $\mu_{F1}=7t$, and $\mu_{F2}=2t$, and standard deviations of $\sigma_{T1}=\sigma_{T2}=6t$, $\sigma_{T2}=1.5t$, $\sigma_{T4}=\sigma_{T5}=3t$, $\sigma_{F1}=2.1t$, and $\sigma_{F2}=0.6t$.

Observe that the performance functions of the individual failure modes of both of these nonseries systems involve the maximum and minimum functions of the component's properties, and therefore are not smooth functions.

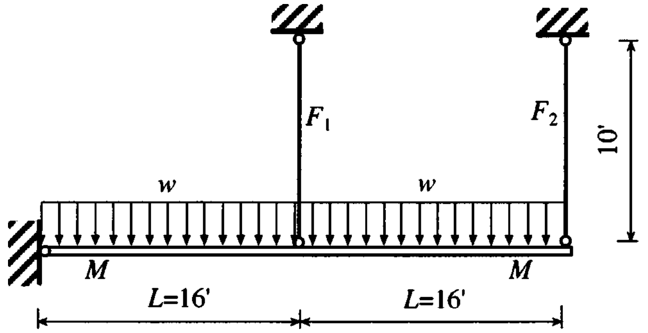

Fig. 6. Beam-cable system

For Eqs. (33) of the parallel-chain system, the first four moments of G^* are approximately $\mu_G=993.3599$, $\sigma_G=316.836$, $\alpha_{3G}=-0.275$, and $\alpha_{4G}=3.109$. With these the first four moments of the performance function, the moment-based reliability indices are as follows:

$$\beta_{2M}=3.135,$$

$$\beta_{3M}=2.802 \text{ with } P_F=2.543\times10^{-3}$$

$$\beta_{4M}=2.818 \text{ with } P_F=2.413\times10^{-3}$$

MCS with 500,000 samples yield the probability of failure for this system as 2.506×10^{-3} with a corresponding reliability index of $\beta=2.806$. The COV of this MCS estimate is 2.82%. One can see that both the results of the third and fourth moment approximations are in close agreement with the MCS results, whereas the second-moment approximation overestimated the reliability index by 12%.

For Eqs. (34) of the truss system, the first four moments of G^* are $\mu_G=8.972$, $\sigma_G=3.732$, $\alpha_{3G}=-0.165$, and $\alpha_{4G}=3.752$. Finally, with these first four moments of the performance function, the moment-based results are as follows:

$$\beta_{2M}=2.404$$

$$\beta_{3M}=2.285 \text{ with } P_F=0.0111$$

$$\beta_{4M}=2.226 \text{ with } P_F=0.0130$$

MCS [performed independent of Eq. (34), i.e., by considering all the possible sequences of component failures that can lead to system failure] with 500,000 samples, gives the probability of failure for this system as 0.0139 with a corresponding reliability index of $\beta=2.199$. The COV of this MCS estimate is 1.19%.

Example 5: Beam-Cable System

Consider the simple elastoplastic beam-cable system shown in Fig. 6 (after work by Ang and Tang 1984). The performance functions of the potential failure modes are listed below with the respective FORM reliability indices indicated in parentheses:

$$g_1=6M-L^2/2 \quad (\beta_F=3.322) \tag{35a}$$

$$g_2=F_1L+2F_2L-2wL^2 \quad (\beta_F=3.647) \tag{35b}$$

$$g_3=M+F_2L-wL^2/2 \quad (\beta_F=4.515) \tag{35c}$$

$$g_4=2M+F_1L-wL^2 \quad (\beta_F=4.515) \tag{35d}$$

where M, F_1, F_2, and w are normally distributed with mean deviations of $\mu_w=2$ kip/ft, $\mu_{F1}=60$ kip, $\mu_{F2}=30$ kip, and $\mu_M=100$ ft/kip, and COVs of $V_w=0.2$ and $V_F=V_M=0.1$.

Using the five-point estimate, the first four moments of G^* are $\mu_G=293.432$, $\sigma_G=76.352$, $\alpha_{3G}=-0.574$, and $\alpha_{4G}=3.265$. Finally, with the first four moments of the performance function, the moment-based results are as follows:

$$\beta_{2M}=3.843$$

$$\beta_{3M}=2.976 \text{ with } P_F=1.460\times10^{-3}$$

$$\beta_{4M}=3.170 \text{ with } P_F=7.634\times10^{-4}$$

MCS with 500,000 samples gives the probability of failure for this system as 5.26×10^{-4} with corresponding reliability index of $\beta=3.276$. The COV of this MCS estimate is 6.16%. One can see that the $2M$ reliability index contains a significant number of errors (17%) and the $3M$ reliability index underestimates the reliability index by 9.2%, whereas the $4M$ reliability index has about 3.2% error which is still large. Correctly, using Eqs. (6) and (8), the five-point estimates for the first four moments are $\mu_G=297.270$, $\sigma_G=78.780$, $\alpha_{3G}=-0.2089$, and $\alpha_{4G}=3.2546$. With these more accurate first four moments of the performance function, the $2M$ reliability index is $\beta_{2M}=3.733$ and the $3M$ and $4M$ reliability indices are $\beta_{3M}=3.315$ and $\beta_{4M}=3.255$. Clearly, with these latter results, the third- and fourth-moment approximations are now in closer agreement with the MCS results. This means that if the first four moments are correctly obtained, the reliability of a system can be computed without only significant error. As illustrated in this example, the approximation of the system performance function using Eqs. (9) and the moments generated with Eqs. (10) may, in rare cases, contain significant error. In this case, Eqs. (7) and (8) may be required in order to obtain more accurate results for the moments.

Principal Conclusions

1. A moment-based method for assessing the system reliability of series and nonseries structures was proposed, with emphasis on series systems. The method directly calculates the reliability indices (and associated failure probability) based on the first few moments of the system performance function of a structure. It does not require a reliability analysis of individual failure modes; also, it does not need iterative computation of derivatives, nor computation of mutual correlations among failure modes, and does not require any design points. Thus, the moment method proposed should be more effective for evaluation of the system reliability of complex structures than currently available computational (non-MCS) methods.
2. The method also includes the approximate system performance function, $G^*(\mathbf{X})$ of Eqs. (9) and the first four moments of Eqs. (10), both of which lead to significant simplification of the calculation of system reliability indices.
3. The accuracy of the results obtained with the proposed method was thoroughly examined by comparisons with large sample Monte Carlo simulations. The fourth-moment reliability index for a structural system is invariably close to the corresponding reliability index obtained from large sample Monte Carlo simulations. The error associated with the third-moment reliability index may be acceptable for practical purposes, whereas the second-moment reliability index invariably leads to very significant unacceptable errors.
4. The accuracy of the $4M$ reliability index inferred above is generally limited to problems with not very small failure probabilities. However, for problems that involve extremely small failure probabilities, such as the final case examined in

example 2, the first four moments may not be sufficient; in such cases, higher-order moments would be required for accurate results which would necessarily entail more complicated calculations.

5. There may be occasions, such as that in the case examined in example 5, when the exact system performance function and associated moments of Eqs. (7) and (8) may be required for more accurate results. Nevertheless, for realistic and practical structures, the $3M$ and $4M$ reliability indices obtained by simplifying Eqs. (9) and (10) should be sufficiently accurate for overall assessment of system reliability.

Acknowledgments

This work was performed while the first writer was visiting the University of California, Irvine, in 2001–2002 as a visiting researcher supported by the Dept. of Education and Science of Japan. This support is gratefully acknowledged. The critical comments and constructive suggestions of Professor A. Der Kiureghian, University of California, Berkeley, are highly appreciated.

References

Ang, A. H.-S., and Ma, H. F. (1981). ''On the reliability of structural systems.'' *Proc., 3rd Int. Conf. on Structural Safety and Reliability*, 295–314.

Ang, A. H.-S., and Tang, W. (1984). *Probability concepts in engineering planning and design*, Vol. II Decision, Risk, and Reliability, Wiley, New York.

Bennett, R. M., and Ang, A. H.-S. (1986). ''Formulation of structural system reliability.'' *J. Eng. Mech.*, 112(11), 1135–1151.

Cornell, C. A. (1966). ''Bounds on the reliability of structural systems.'' *J. Struct. Div. ASCE*, 93(1), 171–200.

Ditlevsen, O. (1979). ''Narrow reliability bounds for structural systems.'' *J. Struct. Mech.*, 7, 453–72.

Grigoriu, M. (1983). ''Approximate analysis of complex reliability problems.'' *Struct. Safety*, 1, 277–288.

Grigoriu, M., and Lind, N. C. (1980). ''Optimal estimation of convolution integrals.'' *J. Eng. Mech. Div.*, 106(6), 1349–1364.

Hong, H. P. (1996). ''Point-estimate moment-based reliability analysis.'' *Civ. Eng. Sys.*, 13, 281–294.

Moses, F. (1982). ''System reliability developments in structural engineering.'' *Struct. Safety*, 1, 3–13.

Ono, T., Idota, H., and Dozuka, A. (1990). ''Reliability evaluation of structural systems using higher-order moment standardization technique.'' *J. Struct. Constr. Eng., AIJ*, 418, 71–79 (in Japanese).

Parkinson, D. B. (1978). ''First-order reliability analysis employing translation systems.'' *Eng. Struct.*, 1, 31–40.

Pearson, E. S., Johnson, N. L., and Burr, I. W. (1979). ''Comparison of the percentage points of distributions with the same first four moments, chosen from eight different systems of frequency curves.'' *Commun. Stat.-Simul. Comput.*, 8, 191–229.

Stuart, A., and Ord, J. K. (1987). *Kendall's advanced theory of statistics*, Griffin, London, 1, 210–275.

Thoft-Christensen, P., and Murotsu, Y. (1986). *Application of structural system reliability theory*, Springer, Berlin.

Tichy, M. (1994). ''First-order third-moment reliability method.'' *Struct. Safety*, 16, 189–200.

Winterstein, S. R. (1988). ''Nonlinear vibration models for extremes and fatigue.'' *J. Eng. Mech.*, 114(10), 1769–1787.

Zhao, Y. G., and Ono, T. (2000a). ''New point estimates for probability moments.'' *J. Eng. Mech.*, 126(4), 433–436.

Zhao, Y. G., and Ono, T. (2000b). ''Third-moment standardization for structural reliability analysis.'' *J. Struct. Eng.*, 126(6), 724–732.

Zhao, Y. G., and Ono, T. (2001). ''Moment methods for structural reliability.'' *Struct. Safety*, 23(1), 47–75.