

Developing a Pedestrian Agent Model for Analyzing an Overpass accident

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Recent softwares have enabled us to apply pedestrian dynamics models into analyses on pedestrian accidents. The author had already developed a pedestrian dynamics model, called ASPF, based on the cell space model, that is evolved from Cellular-Automata (CA). After a couple of years' efforts for continuous revision, we analyze the causes on an accident of Asagiri Pedestrian Overpass in 2001 even though retrospectively. In this paper, the typical existing models for pedestrian dynamics are reviewed, especially explaining on the class of cell space models. Next, ASPF (Agent Simulator of Pedestrian Flows) are explained, in that each pedestrian moves according to several behavioural rules on the cell-grid space of 40 cm side each. Based on not only the fundamental findings from the existing spatial researches but also these from the accident report, ASPF ver.2 is 'tuned up' carefully. ASPF ver.2 is to assess measures for managing pedestrian flows by focusing on the domino risk (density 3 to 5) that shows a symptom for the accident rather than a reconstruction of the real accident itself that had occurred at extremely high density (more than 10). The simulation results show that a two-way flow, combined with standing spectators (stoppers) can trigger an accident even on an overpass that satisfies present design standards. Moreover, we have confirmed that even simple traffic regulations such as partitions can be an effective measure to prevent a pedestrian accident.

1. Introduction

Recent softwares have enabled us to apply pedestrian dynamics models into analyses of pedestrian accidents. The author had already developed a pedestrian dynamics model, called ASPF, based on the cell space model, that is evolved from Cellular-Automata (CA). After a couple of years' efforts for continuous revision, we analyze the causes on an accident of Asagiri Pedestrian Overpass in 2001, even though retrospectively.

This paper addresses our development project of a pedestrian agent simulation model, ASPF. In section 2, after a bottom-up approach is described, the cell space models on which our pedestrian model is based and which belong to a class of the models are reviewed with typical existing models for pedestrian dynamics. ASPF (Agent Simulator of Pedestrian Flows) is also explained, in that each pedestrian moves according to behavioural rules on the cell-grid space of 40 cm side each. In section 3, the development circumstances, the outline of ASPF ver.2 that tried an accident analysis, the results of the simulation analysis and consideration are described in due order. Based on not only the fundamental findings from the existing spatial researches but also these from the accident report, the rule configuration of ASPF ver.2 has been 'tuned up' carefully. ASPF ver.2 is to assess measures for managing pedestrian flows by focusing on the domino risk (density 3 to 5) that shows a symptom for the accident rather than a reconstruction of the real accident situation itself that had occurred at extremely high density (more

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than 10). The simulation results show that a two-way flow, combined with standing spectators can trigger an accident even on an overpass that satisfies present design standards. Moreover, we have confirmed that even simple traffic regulations such as partitions can be an effective measure to prevent a pedestrian accident. Instead of conclusion, our recent revise works of ASPF and future works are to be stated.

2. Existing Researches and Our Approach

2.1. Bottom-up approach

Pedestrian agent modelling appears to be under developed when compared to vehicle modelling, partly because the micro behaviours of pedestrians are so richly varied. The author, who has an interest in the practical application of modelling from the viewpoint of urban planning, can perceive a bottom-up modelling approach – loading higher-order functions for a pedestrian agent on the cell space in order – as this process is developed, it would be cross checked against survey data and observation gained through research.

Firstly, to explain this bottom-up approach, the broad body of existing research has been divided into four categories (Fig. 1). The first category is the actual survey, observation and measurement of pedestrian behaviour and pedestrian flow¹⁻⁷. In this category, a wide variety of case studies have been accumulated; taking fundamental data through to practical application, from micro to macro, from the every day's behaviours to panic behaviours.

The second category is research on models of pedestrian dynamics demonstrating the emergence of macro phenomena in pedestrian flow from the accumulation of micro-motives of pedestrians' behaviours⁹⁻²¹. This category includes the physical phenomenon analogy model, the CA model and the cell space model evolved from CA. However, in many cases, pedestrians just make straightforward movements and avoid others.

The bottom-up approach means the development of a model that loads such higher-order functions as route selection, trip planning and the scheduling of a visiting order into this second category of pedestrian model; all of which must be checked against data taken from survey research.

The third category is research on elemental models of these functions²²⁻²⁵. Much of this research has been studied in OR, transportation planning or the artificial intelligence fields.

The fourth and final category is research to study integrated models for application incorporating all these models by using platforms such as GIS²⁶⁻²⁹.

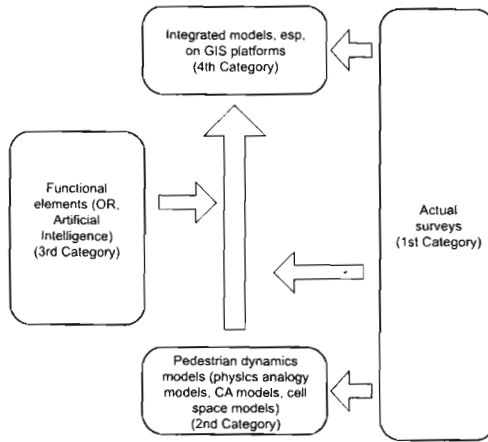


Figure 1: Existing Studies and the bottom-up approach

2.2. From CA model to cell space model

The key characteristic that distinguishes pedestrian behaviour from the physical phenomenon analogy describing the dynamic movement of gas, liquid and granules is avoidance behaviour; humans unlike particles naturally move out of each others way. By considering this characteristic, models called pedestrian dynamics were created; very different models to the physical phenomenon models.

In the theoretical model by Helbing, an acceleration vector of a pedestrian is expressed by the sum of social force and the individual term, and the social force is also expressed by the sum of the accelerating force, border repulsive force, interaction with other pedestrians and time term sums¹². Helbing sought this idea in the concept of “field” in social science. In this connection, Burstedde’s model uses the same concept of field used in the cell space model that will be mentioned later. In this model, the selection probability for each forward movement cell is in proportion to the product of two potentials in that place; the static field and the dynamic field. In particular, the dynamic field is interpreted in a way that treats the other pedestrians’ walking history as a pheromone¹⁰. Now, the CA model and its extended type – the cell space agent model, or more simply the cell space model – are described starting with a brief explanation of the difference between the two.

Firstly, as an example of a simple two-dimensional CA, the life game is examined. This CA has three characteristics: (1) update structure – the state of cells in a term are all updated according to information from the previous term; (2) neighbourhood – the use of the state of the neighbouring cells as information to prescribe the cell’s update; and (3) two value status – the state of cells is a two value status of light ON or light OFF.

When pedestrian behaviour is expressed using this CA, after assuming that a pedestrian is positioned on a light ON cell, a concept of movement is introduced, expressed by (4) light ON and OFF – as a cell turns its light OFF, another cell turns its light ON. In

this case, (5) the upper limit of occupancy – the maximum number of pedestrians that can occupy one cell is assumed to be one. In the case of a one-way flow, this model can express pedestrian behaviour and for a one-dimensional CA, rule 184 of Wolfram²¹ is applied.

In comparison, when dealing with a multi-directional flow or a long movement with one step, such conditions as update structure, neighbourhood, two value states or upper limit of occupancy need to be eased. In particular, when the update structure is changed, one of the following rules needs to be applied:

- » after updating all cells relating to movement by using information from the previous term in parallel, solve the conflict for cells that have surpassed the upper limit of occupancy;
- » apply an order to every pedestrian and move them in turn.

The former is called a parallel update and the latter a sequential update. The sequential update has a path dependency – a calculation result differs according to an applied order – therefore, some researchers prefer parallel updates and some prefer sequential updates, especially with random sequential updates (RSU), when random numbers are used to establish an order of precedence.

When using these methods, especially in a sequential update, the expression form of a cell space is given as a walking space model, but in a narrow sense this is beyond the definition of CA and it is common to call it an agent model. In this paper this is called a cell space model and is an extension of the CA model. Table.1 shows recent research using this cell space model. All the research is designed to enable the emergence of pedestrian flow macro phenomenon from the accumulation of pedestrian agents' micro behaviour; however, while the two models on the left side are included mainly for theoretical interest, the three models on the right side show the characteristics of model development reproducing macro phenomenon observed by survey research. Particularly, our ASPF ver.2 on the far right also aims at measurement evaluation by comparing to numerical survey data.

When modelling cell spaces, discrete approximation is also important; in the mutual comparison of Table 1, the space scale of a cell is nearly 40 cm including unspecified ones. On the other hand, variations in time scale can be seen, but this variation appears to be related to the distance within which avoidance behaviour starts. Each of the algorithms in the three models on the right side in Table.1 has a different characteristic and they can be classified in the following ways:

- » The model of Burstedde et al. – has a short measurement for both time slice and the avoidance behavior distance, with a focus on high density flows.
- » The model of Blue and Adler – has a short time slice and a long avoidance behaviour distance with a focus on low density flows.
- » The model of the author et al. – has a middle measurement for both and focuses on the changing processes from low to high density flows.

	Models	Muramatsu et.al (1999)	Fukui, Ishibashi (1999)	Burstedde et.al (2001)	Blue, Adler (2001)	ASPF ver.1 (2002) and ver.2 (2004)
Time and Space Scales	Cell Size	No description	No description	40 cm side each	45.7 cm side each	40 cm side each (ver.2)
	Time Slice	No description	No description	approx. 0.3 sec	No description (1 sec)	0.5 sec
	Cell Occupancy Limit	One person	One person	One person	No description (one person)	Two with a surrounding density of two or more, one with other case (ver.2)
	Behavioral Rules, (Number on the rules in bracket)	To move to one of three adjacent cells (forward, right or left), according to the situation of the adjacent three cells	(Diagonal Stepping Mode) To move forward one in a case that the forward interval is one or more (Wolfram's no.184 rule). To avoid to a cell diagonally forward, if possible, in a case of the existance of the other on the adjacent forward cell. Otherwise, to stop	To move to one of the adjacent eight cells, according to these cells. The probability is proportional to M (dynamic potential) and S (static potential). Potential concepts are analogous to the recent physics	To choose the maximum interval lane from forward three lanes, and move forward on it. In cases of same length of interval, use randomness. Adjacent two confronters exchange their position by using randomness.	Basic behaviors (6). Avoiding others (4). Slowdown (8). High density walking (3). Flow cognition (1), Cornering (ver.2)
Simulation Algorithm	Maximum Speed/ Distribution of the maximum Speed	One	One	One	Four 5%, Three 90%, Two 5%	Three
	Maximum interval to avoid/Symmetry	One / Symmetric	One / Symmetric	Two / Symmetric	Eight / Symmetric	Three / asymmetric (a flow cognition rule)
	Updating & Conflict Resolution Methods	SU (Sequential Updating)	No description	PU (Parallel Updating), then conflict resolution by using random numbers	Two times PU at the avoidance and advance	RSU (Random Sequential Updating)
Main Findings of the Study		(1) Emergence of jam in confrontation flows, (2) Calculation of relation between flow coefficient and speed, (3) Case study on characteristics of avoidance behaviours	(1) Proposal of side stepping model and diagonal stepping model, (2) Phase transition from straight-forward to stable diagonal in cases of confrontation flows on DSM, (3) Calculation of a density - speed relation	(1) Representation of refugees' flows in indoor space, (2) Emergence of stable 'lanes' in confrontation flows	(1) Calculation of relations among flow-phases, ratios of confrontation and exchange probabilities, (2) Estimation of an exchange probability, based on cross-check of actual measurement, (3) Calculation of relations of density-frequency of avoidance	(1) Emergence of stratification in confrontation flows (ver.1), (2) Cross-check graphs of density-speed to actual surveys (ver.2), (3) Simulation of an accident situation (under density 4), based on the descriptions of the office ex post facto inspection (ver.2), (4) Calculation of effects of crowd control measures (ver.2)

 Table 1: Comparison among agent models with cell spaces^{17,11,9,8,15,25}

3. Agent Simulation of Pedestrian Flows (ASPF) Ver.2

3.1 Development of ASPF

The author's laboratory had been dealing with the modelling of pedestrians' shopping-around behaviours for some years and early in the summer of 2001, we started to study the modelling of more basic pedestrian flow with the aim of trying out the performance of an agent simulation software¹; in recent years the usefulness of the software has been significantly improving.

In July while designing a model, a tragic pedestrian accident occurred at Akashi City Firework Festival; articles of the event and speculation of the causes were given in the press. These articles became the basis for ASPF ver.1 of the software program which was published in December; the program examined high density pedestrian flow in an L-shaped corridor and was modelled on the Asagiri Station Pedestrian Overpass¹⁵. In January 2002, Akashi City published an accident report – ex-post facto inspection –¹ revealing further detailed information, which was then used to review and improve ASPF ver.1, reconstructed the situations of the accident and led the revision into ASPF ver.2. The simulation analysis of this program was published in February 2004²⁰.

3.2. Simulation setting for an accident analysis

In the following section, an outline of the ASPF ver.2 model is given.

The main components of this agent model are the walking behaviour rules as shown in Fig. 2.

With a threshold density of 2 persons/m², at a low density time the following rules are applied: 6 basic behaviour rules, 8 slow-down rules, 4 avoidance rules and 1 flow pattern cognition rule. At a high density time, one of the 3 high density flow rules is applied. The priority order is a simple numerical order.

For walking on a L-shaped corridor, in addition to a cornering rule, a rule for shortening a route while cornering at a low density time was devised (Fig. 3).

One step was set at 0.5 seconds, 1 cell at 40 cm each and the maximum number of occupants per cell was 2 persons with a surrounding density of 2 or more and 1 person with any other density.

In the measurement of the relationship between the density and flow speed, a graph sloping to the right and closely resembling the survey research data, was obtained (Fig.4). The accident report concluded that the accident had occurred when the density on the overpass reached 13 to 15. On the other hand, typical domino accidents— in Japan, most of that people injured had occurred on the stairs, even that can occur on flat ground—can happen at density 3 to 5. So, in our simulation analysis under ordinary cases, not panic cases, a walking behaviour model in an L-shaped flat space was used where the difference between the stairs were eliminated and at the same time, a density of 4 – congestion giving rise to a possible domino accident – was focused on as the risk level.

It is also important to consider the flow coefficient. The Asagiri Pedestrian Overpass

¹ KK-MAS by Kozo Keikaku Engineering Inc., Japan¹⁶

was designed with an evacuation plan standard of 1.5 person/m • sec, which is far higher than the usual design standard of 0.33 person/m • sec^{5,7}. However, both values assume a one-way flow. The flow coefficient value estimated by the author et al from the report was 0.48 at the peak time; a much lower value than the maximum capacity, but the accident still happened. The accident report states contributory factors to the accident were: 1) two counter flows meeting and becoming tangled up; and 2) fireworks spectators blocking the area around the south part of the pedestrian overpass.

In the simulation analysis on the south side of the L-shaped corridor (overpass: 17 cells wide, stairs: 9 cells wide), standing spectators (stoppers) in several rows of 2 to 5 cells were set up to give a trial calculation for these influences on the increase of density. The accident report also stated that effective crowd control measures were not carried out; the simulation analysis examined the effect of placing a partition in the center to control the crowd (Fig.5).

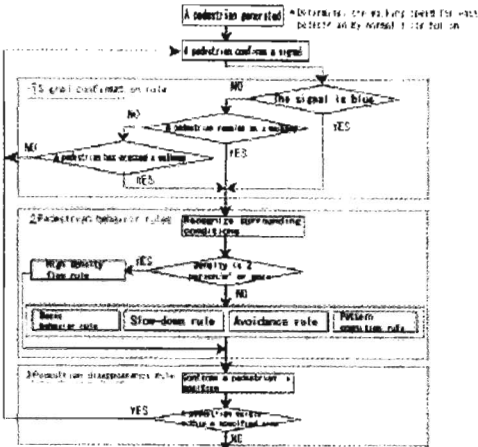


Figure 2: Algorithm of Pedestrian Agent in ASPF ver.2

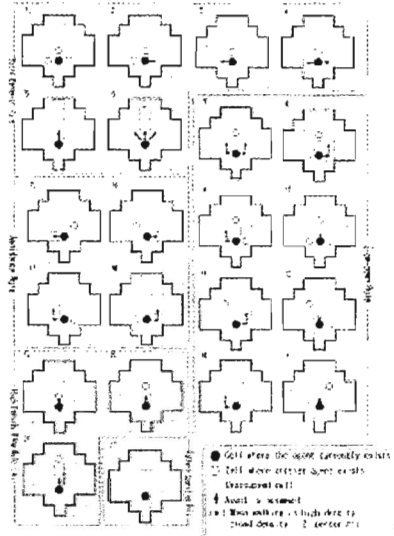


Figure3: Behavioural rules of ASPF ver.2

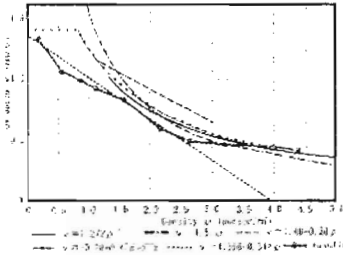


Figure 4: Benchmarking test of agent flows

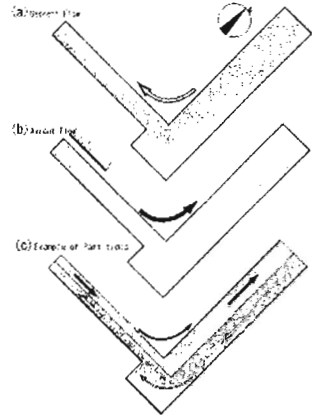


Figure 5: L-shaped model and density distributions

3.3. Results and implications

The simulation result showed that when there were no standing spectators, even with two counter flows, as long as the sum of both flow coefficients was less than 1.5, the density never exceeded the risk level, but when the number of rows of stoppers (standing spectators) was 3 or more, the density increased and exceeded the risk level (Fig.6).

The simulation also showed that the introduction of a simple partition controlled the crowd density, but when the stair width was divided into two parts – each half was 1.8 m; below 2.1 m – this minimum width resulted in a bearing experiment to examine an arch action. In conclusion it was found that the introduction of a one-way flow and a prohibition on standing are the best safety precautions.

Here we will briefly mention about simulation analysis. Computer simulation is known as a tool for ‘forward reasoning’, which draws conclusions from assumptions. However, recent improvements in operational capability have allowed ‘backward reasoning’ to narrow the possible assumptions for a given conclusion, by examining a variety of different conclusions through changing assumptions in the simulation. Generally, in an ex post facto inspection it is considered difficult to gather all the data items necessary for determining the cause. In such a case, a simulation is carried out by using any available data in conjunction with a working hypothesis; this allows a reconstruction of the accident situation and the exploration of a logical coherent cause and effect explanation. This type of simulation analysis is used within this paper and it would appear that the analysis made in this section is significant, as by using a model including information from detailed and consistent accident reports, the accident process was reconstructed, accident data was confirmed to be accurate and moreover it was possible to attempt density calculations and numerical evaluations for the implementation of counter measures.

When conducting this type of analysis, it is important to collect many facts and several models to prove or disprove any hypothesis, and closely scrutinize their relationships. In the case of pedestrian flow, the testimonies of pedestrians relating their experiences are also necessary.

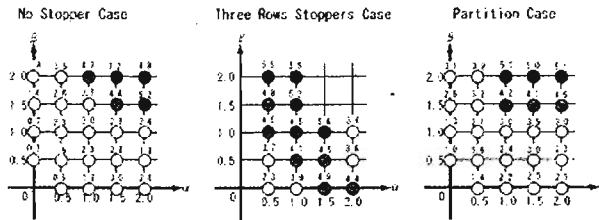


Figure 6: Simulation Results

4. Concluding Remarks—Further Development

This paper explained ASPF ver.2 by which we analyzed a pedestrian accident as an instance of the class of cell space model. ASPF ver.2 has a structural restriction of the cell space mainly caused by the grid shape, when simulating pedestrian’s cornering behavior etc. Fortunately, the software that we use as the platform for developing our ASPF models provides us a modeling environment to make agents’ walking in a free direction. Then the authors have also revised ASPF further -- while the simulator moves each pedestrian agent on XY continuous coordinates space, but inside of each agent, he/she has own relative cell space according to the walking direction and the same behavioral rules as ASPF ver.2’s. By using this ASPF ver.3, we tried a crowd analysis on an ‘scramble’ intersection with straight and diagonal crosswalks (Table.2).

If a free direction walk of an agent can be expressed, simulation of the cornering behavior on a L-shaped corridor will be considered to become still minuter. Therefore, we are going to introduce a direction correction function (Helmsman function²⁹) that corrects the direction to a given walking target (waypoint) periodically even when an agent receives other agents’ interference, and to tackle the accident analysis in the same L-shaped corridor space model as this paper’s analysis again and again. «

	Main revises
ASPF, ver.1 (Dec. 2001) ¹⁵	<ul style="list-style-type: none"> » Was developed as a prototype agent model with 18 rules on a cell space » Had verified an emergence of stratification in confrontation flows » Had tried to simulate an accident process on a L-shaped corridor space
ASP, ver.2 (Feb. 2001) ²⁰	<ul style="list-style-type: none"> » Was revised to 22 rules, based on examination of factual findings » Had simulated the accident process by introducing conering rules » Had assessed partition effects for crowd control
ASPF, ver.3 (Dec. 2004) ¹⁴	<ul style="list-style-type: none"> » Has been implemented proper crusing speed of each agent » Has enabled agent's free-direction walking on relative coordinates system » Has compared with observation at ,scramble' crosswalk
ASPF, ver.4 (Dec. 2005)	<ul style="list-style-type: none"> » To be examined and revised the high-density rules through benchmarking tests » To be introduced Helmsman function instead of the conering rules » To be re-simulated the accident process on L-shaped corridor

Table 2: Version up summary

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