Probabilistic Evaluation of Column Overdesign Factors for Frames

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Abstract: In the earthquake resistance design of frame structures, the yielding of all the beams in flexure prior to possible yielding of columns is generally considered to be a preferable mode; this is defined as a beam-hinging pattern in this paper. Structures are therefore generally weak-beam, strong-column designed with a column overdesign factor (COF). In reality however, the designed weak-beam, strong-column structures may not collapse according to the preferable failure mode because of large uncertainties in external loads and member strengths. These uncertainties may change the COF and the structure may collapse according to some unpreferable failure modes such as the weak story mechanisms. The object of this paper is probabilistic evaluation of the target values of COFs for which the occurrence probability of unpreferable failure modes can be limited to within a specific tolerance. A stochastic limit analysis procedure using the linear programming method and the first-order reliability method is developed, and the likely failure modes of weak-beam, strong-column designed structures are investigated using this procedure. The target value of COF is discussed in probabilistic terms. It is found that a much higher value of COF is required in the case of uncertain loads and member strengths than is expected in deterministic cases.

INTRODUCTION

When carrying out the deterministic design of frame structures, some preferable failure modes often are selected, and the strengths of the structural members are designed according to these selected failure modes. The weak-beam, strong-column designed structure is commonly used in earthquake resistance design to make the frame structure collapse according to the entire failure pattern, which allows the yielding of all the beams in flexure prior to possible yielding of columns (hereafter, it is referred as the beam-hinging pattern). This is considered to be a suitable failure pattern because of its large ability to absorb earthquake energy before the structure actually collapses (Anderson and Gupta 1972; Park and Paulay 1975; Clough and Penzien 1982; Lee 1996).

To ensure that a frame structure collapses according to the beam-hinging pattern, the columns of the structure are generally overdesigned with a column overdesign factor (COF). However, when a frame structure is designed as a weak-beam, strong-column structure, an important consideration is if the designed structure collapses according to the designed failure mode. This problem is caused by large uncertainties in external loads and member strengths. These uncertainties may change the designed COF, and the structure may collapse according to some unpreferable failure modes such as the partially column-failure pattern.

To introduce uncertainties in external loads and member strengths into structural design, it is necessary to investigate all or at least some of the more important failure modes that result from these uncertainties. However, the number of potential failure modes is generally too large, and the search for important failure modes may be complicated. It is almost impossible for a design engineer to consider such a large number of failure modes and their occurrence probabilities. It is a general requirement for researchers to give a suitable target value of COF for which the occurrence probability of unpreferable failure modes can be limited to within a specific tolerance.

Some case studies have been conducted and some basic knowledge has been obtained on the value of COF that results in the beam-hinging pattern mode (Kuwamura et al. 1989; Kawano et al. 1998). However, almost all the previous studies investigated only some specific examples, because the analytical models used were too complicated. General investigation is needed.

The object of this paper is to discuss the target COF for such a general case. A stochastic limit analysis procedure using the linear programming method and the first-order reliability method (FORM) is developed. Then, using this procedure, the likely failure modes of weak-beam, strong-column designed structures under uncertain loads and with uncertain member strengths are investigated, and the target value of COF required to ensure that frame structures collapse according to the beam-hinging pattern is discussed.

STOCHASTIC LIMIT ANALYSIS

Computational Assumptions

For the ductile frame structures considered in this study, several commonly used assumptions are applied:

- Elastic-plastic frame structures are considered. The failure of a section means the imposition of a hinge and an artificial moment at this section.
- The structural uncertainties are represented by considering only moment capacities as random variables.
- Geometrical second-order and shear effects are neglected. The effect of axial forces on the reduction of moment capacities is also neglected.

Stochastic Limit Analysis Combined with FORM

The search for computationally efficient procedures for identifying significant failure modes has resulted in many approaches such as the truncated enumeration method (Murotsu et al. 1984; Melchers and Tang 1985; Xiao and Mahadeven 1994) and mathematical programming techniques (Ma and Ang 1981; Nafday et al. 1988a,b; Ellis et al. 1991; Ohi 1991). In this paper, the stochastic limit analysis (Ohi 1991), which is one of the mathematical programming techniques, is used because the likely failure modes can be obtained in relatively short computation time.

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Based on the upper-bound theorem of plasticity (Livesley 1976), failure of a ductile frame is defined as the formation of a kinematically admissible mechanism due to the formation of plastic hinges at a certain number of sections. The mechanism can be identified from the structural analysis when the stiffness matrix becomes singular. The compact procedure (CP) (Aoyama and Kamimura 1988) of limit analysis is used in this paper. In this procedure, the equilibrium equation is taken to be the object function and the ultimate strength is taken as the limit condition. The limit analysis is defined as the problem of obtaining, using the linear programming method, the maximum load factor that satisfies the equilibrium equation and the limit condition. This equilibrium equation is

$$\lambda[E]\{P\} = [H]\{r\} \tag{1}$$

where λ = load factor; $[E]$ = unit matrix; $\{P\}$ = load vector; $[H]$ = coefficient matrix for the equilibrium equation; and $\{r\}$ = vector of member strength.

Applying the Gauss-Jordan method to (1), some columns of $[H]$ will become fundamental columns in which only one element becomes 1 and others become 0. Then the following two steps are repeated until the load factor reaches its maximum:

1. Divide $\{r\}$ into fundamental variables (those corresponding to 1 in the fundamental columns of $[H]$) and nonfundamental variables according to the contents of $[H]$. Change the fundamental variables; increase the load factor until the utmost value (moment capacity) of a fundamental variable is reached.
2. To increase the load factor further, exchange the fundamental variables and the nonfundamental variables.

The above CP is conducted at first using the mean values of the load and member strength. By doing this, some failure modes will be obtained. The performance function corresponding to each failure mode can be obtained readily using the principle of virtual work, and the design point and reliability index for each mode are evaluated by FORM. For the failure modes that have a smaller reliability index, such as those for which the condition $\beta \le \beta_{\min} + \delta$ is satisfied, the CP is conducted again—this time using the design point as a deterministic value of the loads and member strengths. Here, $\beta_{\min}$ is the minimum value of β corresponding to the obtained failure modes; the modes that have reliability index $\beta > \beta_{\min} + \delta$ will be truncated. In this paper δ is taken to be 1. If the load factor λ obtained in the procedure does not reach 1, which means the mean load is larger than the utmost load, then the design point is in the failure area and will be removed from the computer's memory; otherwise, the design point and reliability index are evaluated again for the obtained failure modes. The iteration is continued until no new failure modes are obtained. The modes that appeared in all the iterations are then the likely failure modes of the frame structure. Using this method, the likely failure modes considering nonnormal random variables can be obtained with few iterations. The obtained failure modes have been elaborated by Monte Carlo simulation of limit analysis (Yoshihara 1997).

EVALUATION METHOD OF COF

Basic Assumption in Evaluation

In the results of stochastic limit analysis, the likely failure modes of a specific structure under a specific load are mainly dependent on the mean value and coefficient of variation of the member strength. Because a specific structure is generally constructed using the same kind of material (generally steel or concrete) through the whole structure, and the coefficient of variation is mainly dependent to the material, the coefficient of variation of each member strength was assumed to be the same for all the members in the structure. The mean values of the member strengths therefore can be used as the main factor in the investigation.

For one-story, one-bay structures, COF is defined simply as the ratio between the mean value of the column strength and the mean value of the beam strength. This is because there is only one beam and one column at each node for such structures

$$\mathrm{COF} = \frac{M_{pc}}{M_{pb}} \tag{2}$$

where M_{pc}, M_{pb} = mean values of the ultimate moment of the columns and beams, respectively.

For multistory, multibay structures, because the number of beams or columns is different for each node, COF used in the investigation is defined for each node as the ratio between the sum of the mean values of the column strengths and the sum of mean values of the beam strengths at that node, as follows:

$$\mathrm{COF}(k) = \frac{\sum_i M_{pci}}{\sum_i M_{pbi}} \tag{3}$$

where k = kth node; and M_{pci}, M_{pbi} = mean value of the ultimate moment of the columns and beams, respectively, connected to the kth node.

For convenience of investigation, the following assumptions are applied:

- All the beams and columns are designed to make the structure have the same value of COF at every node, i.e., there is only one value of COF for a specific structure.
- The external load is considered to consist of only the static lateral earthquake loads, which is often used for simple seismic design. These are assumed to be concentrated forces triangularly distributed along the height of the structures in consideration.
- Plastic moment capacities of sections are statistically independent of the applied loads and independent of each other.

All the variables are assumed to have a lognormal distribution. Because the frames in consideration are steel, the coefficient of variation of the member strengths is taken to be 0.1 (*Ultimate* 1990). Because the coefficient of variation for ground motion is generally taken to be 0.6–0.7 (Kanda 1993; *Recommendations* 1993), and considering the other uncertainties included in load modeling, the coefficient of variation for the lateral forces is assumed to be 0.8.

Evaluation Method

If the value of COF is given for a specific frame structure, the likely collapse failure modes can be obtained using the stochastic limit analysis procedure described above. Because the structure is deterministically designed to collapse according to the entire beam-failure mode, the most likely failure mode is generally the preferable beam-hinging pattern mode, and all the other likely failure modes are unpreferable failure modes. Because the second most likely failure mode has the largest occurrence probability among all these unpreferable failure modes, the following evaluation index is used in this paper to evaluate the relative occurrence rate of the unpreferable failure modes:

$$\gamma = \frac{p_{f2}}{p_{f1}} \tag{4}$$

Here p_{f1} = occurrence probability of the most likely failure mode, i.e., the beam-hinging pattern mode; and p_{f2} = occurrence probability of the second most likely failure mode, i.e., the most likely failure mode among all the unpreferable failure modes.

To ensure that the designed structure collapses according to the designed preferable failure mode, the relative occurrence rate of the unpreferable failure modes γ should be limited to within a specific allowable level γ_0 as follows:

$$\gamma = \frac{p_{f2}}{p_{f1}} \leq \gamma_0 \qquad (5)$$

The larger the value of COF, the smaller the value of the relative occurrence rate of the unpreferable failure modes. By conducting stochastic limit analysis using different COFs for a frame structure, a γ-COF curve can be obtained, and the target value of COF for which (5) is satisfied can be determined.

The tolerance level γ_0 should be determined in advance. However, the value of γ_0 that should be set is ambiguous because we do not know clearly what value will be acceptable to designers. In this paper, two levels of tolerance—γ_0 = 0.8 and 0.9—are investigated provisionally.

For a specific value of COF, because γ is affected greatly by the load level (Ono and Zhao 1998), γ should be investigated under different load levels. To consider this effect, two levels of reliability index—β = 2 and 3—are used, and the load levels are designed according to the assumed reliability levels.

Evaluation Example

As an example of the application of the evaluation method described above, COF value for a three-story, two-bay frame structure shown in Fig. 1 was evaluated. For COF = 1.1, the member strengths are designed as listed in Table 1. For other values of COF, the member strengths can be similarly designed

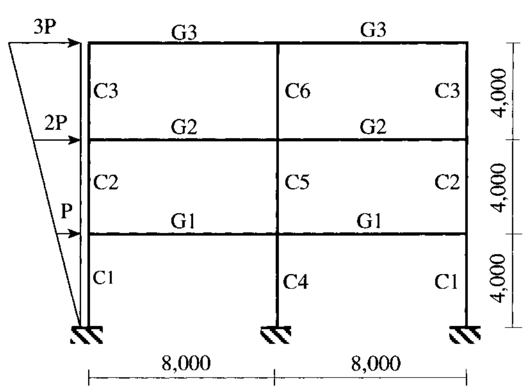


FIG. 1. Three-Story, Two-Bay Frame Structure

TABLE 1. Member Strengths of Three-Story, Two-Bay Structure

Member (1)	Mean value of strength (t cm) (2)	Coefficient of variation (3)
C1	1,687	0.1
C2	1,687	0.1
C3	1,687	0.1
C4	3,374	0.1
C5	3,374	0.1
C6	3,374	0.1
G1	3,068	0.1
G2	3,068	0.1
G3	1,534	0.1

proportional to those in Table 1. According to the results of stochastic limit analysis, the beam-hinging pattern mode first appears when the value of COF $\geq$1.3. The first four likely failure modes for COF = 1.3 are shown in Fig. 2 along with their corresponding first-order reliability index—β = 2.000, β_2 = 2.020, β_3 = 2.035, and β_4 = 2.039. It can be seen from this figure that the unpreferable failure modes (corresponding to β_2, β_3, and β_4) are only slightly different from the beam-hinging pattern (corresponding to β_1).

To conduct COF evaluation at the same reliability level, the external load level was adjusted to ensure that the reliability index corresponding to the most likely failure mode (entire beam-failure pattern) of the structure kept the same value

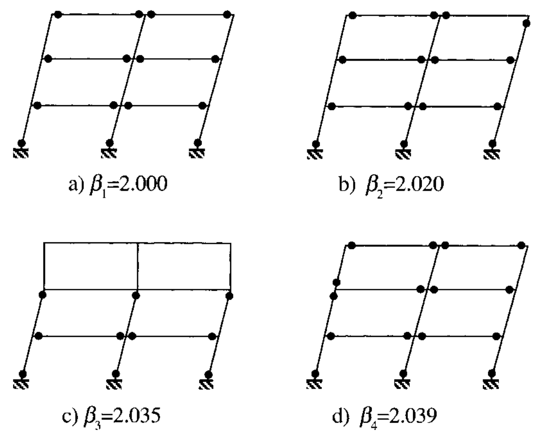


FIG. 2. Four Most Likely Failure Modes of Three-Story, Two-Bay Structure

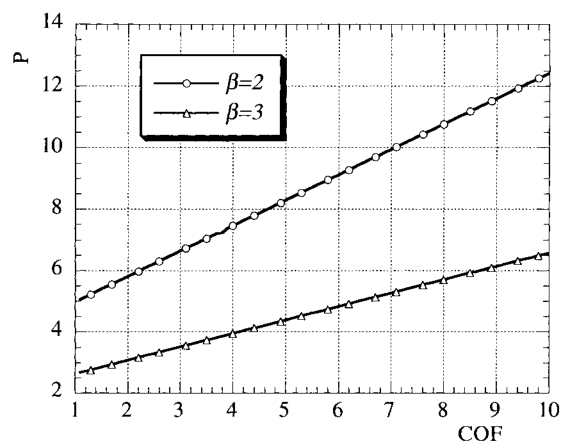


FIG. 3. External Load Corresponding to COF

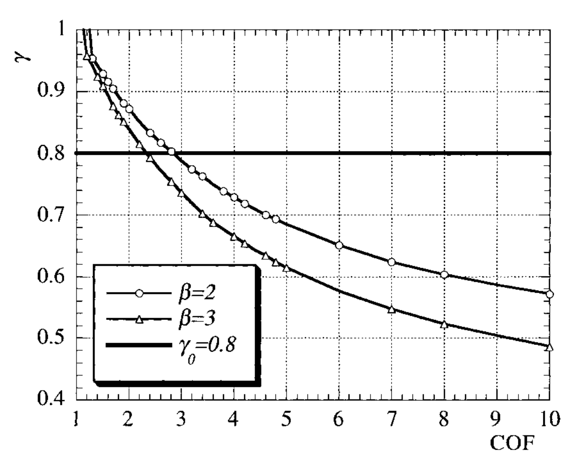


FIG. 4. γ-COF Curve for Three-Story, Two-Bay Structure

when the value of COF was changed. The designed load levels for the two reliability levels, $\beta = 2$ and 3, are depicted in Fig. 3, from which one can see that the load level increases almost proportionally as COF increases.

If the stochastic limit analysis is conducted on the designed frame structure with different values of COF, select the first two most likely failure modes and calculate the probability ratio γ as in (4). Then the two γ-COF curves shown in Fig. 4 for $\beta = 2$ and 3 can be obtained. From Fig. 4, the target value of COF corresponding to the tolerance level $\gamma_0 = 0.8$ can be obtained as 2.83 and 2.33 for $\beta = 2$ and 3, respectively. One can see that the target value of COF for $\beta = 3$ is much smaller than that for $\beta = 2$. For the sake of comparison, target values of COF for $\gamma_0 = 0.7$ were also obtained as 4.61 and 3.44 for $\beta = 2$ and 3, respectively. One can see from this that the target value of COF is very sensitive to the tolerance level γ_0.

PROBABILISTIC EVALUATION OF COF

COF Evaluation for One-Story, Multibay Structures

To investigate COF values of one-story structures, the height-span ratio is taken as H/L = 4 m/8 m = 0.5. The mean strength of the beams are taken to be 1,062 t cm, and the mean strength of the columns changed according to changes in COF. Structures from one to five bays were investigated, and the γ-COF curves for each structure are depicted in Fig. 5 for β = 2 and in Fig. 6 for $\beta = 3$.

From Figs. 5 and 6, one can see that the larger the number of bays, the more gentle will be the slope of the γ-COF curve. For $\gamma_0 = 0.8$, the target values of COF for $\beta = 2$ are obtained as 1.37, 1.74, 2.36, 3.32, and 5.07 for one-, two-, three-, four-, and five-bay structures, respectively; those for $\beta = 3$ are obtained as 1.22, 1.49, 1.83, 2.28, and 2.92, respectively. One can see that the larger the number of bays, the larger the needed target value of COF.

COF Evaluation for Multistory, One-Bay Structures

To investigate COF values of one-bay structures, the height-span ratio is taken as H/L = 4 m/8 m = 0.5. The mean strength of the beam at the top floor is taken to be 1,062 t cm, whereas those of other floors are taken to be 2,124 t cm. The mean

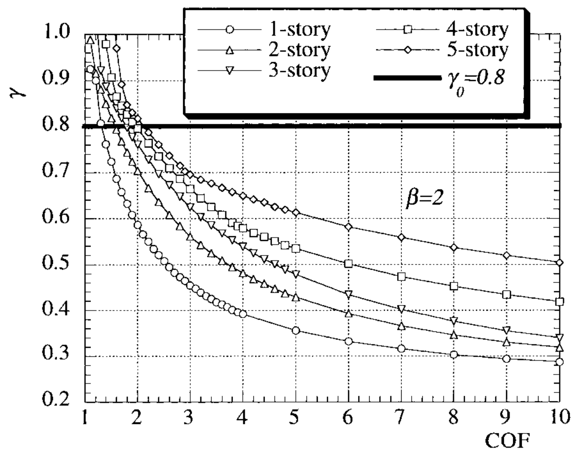


FIG. 7. γ-COF Curve for Multistory, One-Bay Structure ($\beta = 2$)

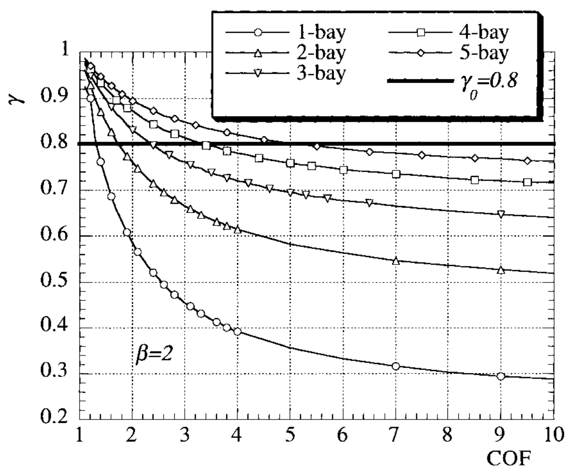


FIG. 5. γ-COF Curve for One-Story, Multibay Structure ($\beta = 2$)

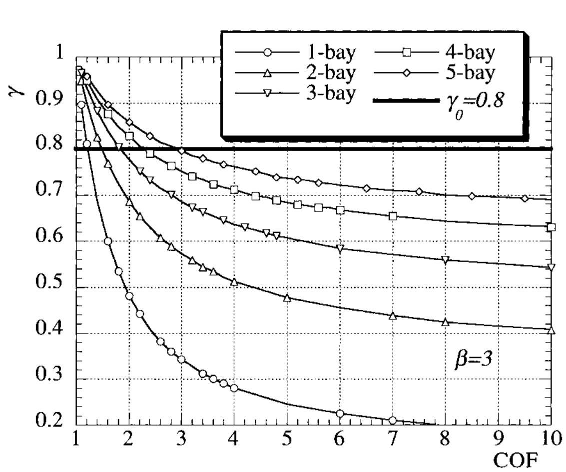


FIG. 6. γ-COF Curve for One-Story, Multibay Structure ($\beta = 3$)

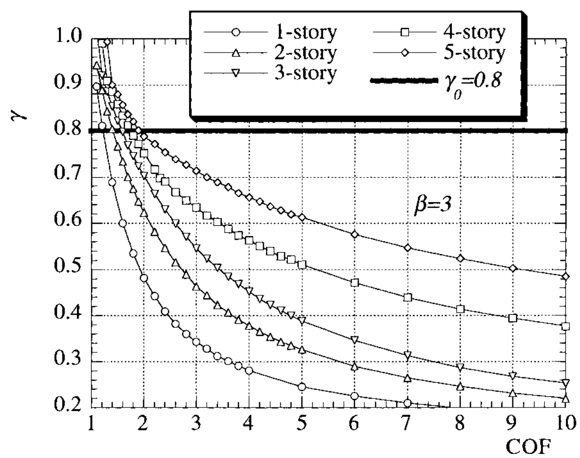


FIG. 8. γ-COF Curve for Multistory, One-Bay Structure ($\beta = 3$)

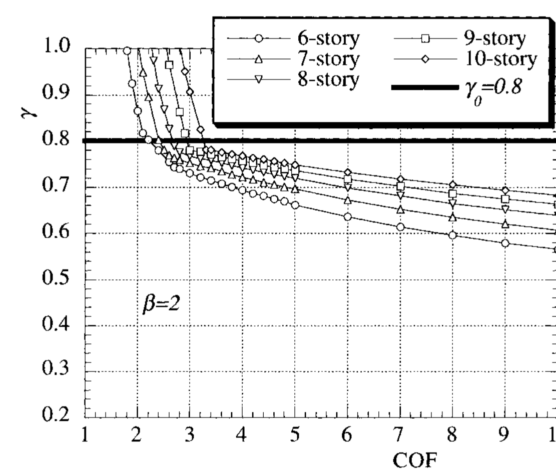


FIG. 9. γ-COF Curve for Multistory, One-Bay Structure ($\beta = 2$)

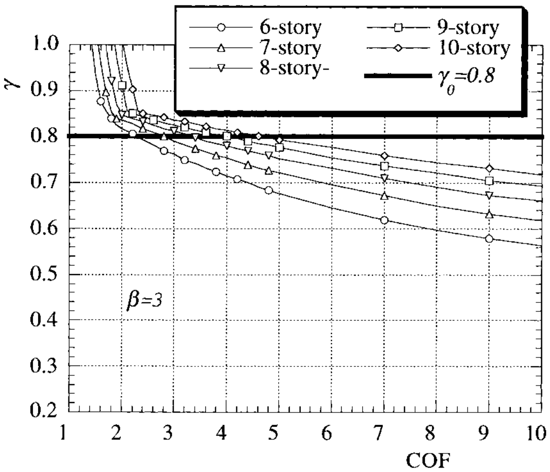
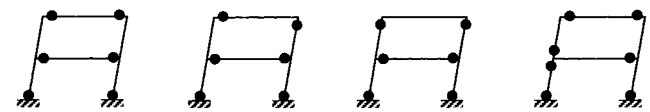
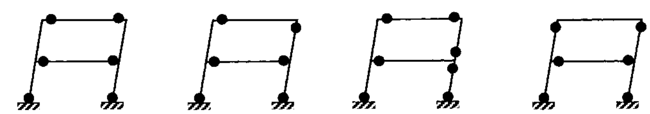


FIG. 10. γ-COF Curve for Multistory, One-Bay Structure (β = 3)



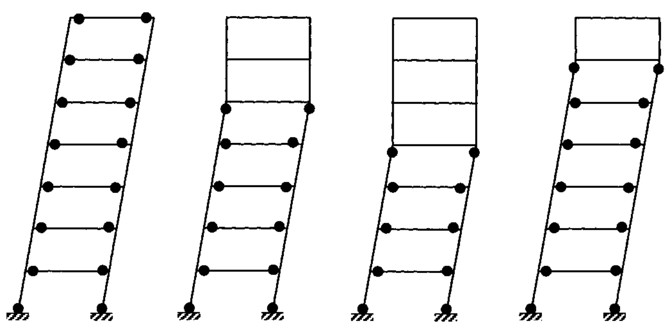
a) β_1=2.0 b) β_2=2.005 c) β_3=2.020 d) β_4=2.038

FIG. 11. Four Most Likely Failure Modes of Two-Story, One-Bay Structure (for COF = 1.1)



a) β_1=2.0 b) β_2=2.336 c) β_3=2.605 d) β_4=2.607

FIG. 12. Four Most Likely Failure Modes of Two-Story, One-Bay Structure (for COF = 5.0)

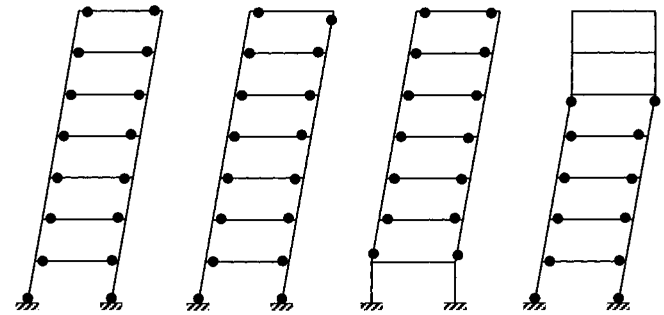


a) β_1=2.0 b) β_2=2.022 c) β_3=2.042 d) β_4=2.065

FIG. 13. Four Most Likely Failure Modes of Seven-Story, One-Bay Structure (for COF = 2.1)

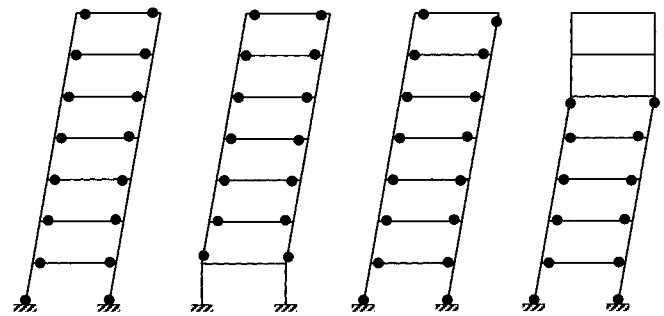
strength of the columns changed according to changes in COF. Structures from 1 to 10 stories were investigated. For the structures from one to five stories, the γ-COF curves are depicted in Fig. 7 for β = 2 and in Fig. 8 for β = 3. For the structures from 6 to 10 stories, the γ-COF curves are depicted in Fig. 9 for β = 2 and in Fig. 10 for β = 3.

From Figs. 7 and 8, one can see that, once again, the larger the number of stories, the more gentle will be the slope of the γ-COF curve. For γ_0 = 0.8, the target values of COF for β = 2 are obtained as 1.32, 1.57, 1.79, 1.97, and 2.12 for one-, two-, three-, four-, and five-bay structures, respectively; those for β = 3 are obtained as 1.23, 1.41, 1.59, 1.75, and 1.91, respectively. One can see that, once again, the larger the number of stories, the larger will be the target value of COF. The change in the target value of COF with increase in the number



a) β_1=2.0 b) β_2=2.102 c) β_3=2.107 d) β_4=2.114

FIG. 14. Four Most Likely Failure Modes of Seven-Story, One-Bay Structure (for COF = 2.5)



a) β_1=2.0 b) β_2=2.111 c) β_3=2.114 d) β_4=2.156

FIG. 15. Four Most Likely Failure Modes of Seven-Story, One-Bay Structure (for COF = 2.7)

of stories is much smaller than that with the increase of the number of bays in the case of one-story, multibay structures.

From Figs. 9 and 10, one can see that there are several discontinuities in the γ-COF curves. Such discontinuities occur at values of COF where the second most likely failure mode changes. For two-story, one-bay structures, the four most likely failure modes for COF = 1.2 and 5.0 are shown in Figs. 11 and 12, respectively. The pattern and order of the two most likely modes remains unchanged as COF increases. Therefore, there are no discontinuities in the γ-COF curve similar to the ones shown in Figs. 7 and 8. For seven-story, one-bay structures, the first four likely failure modes for COF = 2.1 are shown in Fig. 13, and those for COF = 2.5 and 2.7 are shown in Figs. 14 and 15, respectively. The beam-hinging pattern mode appearing as the most likely failure mode for COF is 2.1 or above. The pattern of the second most likely failure mode changed for the cases COF = 2.5 and 2.7; and when COF is larger than 2.7, the pattern of the second most likely failure mode remained unchanged. The change in the second most likely failure mode for the cases COF = 2.5 and 2.7 causes discontinuity in the γ-COF curve at the two points shown in Fig. 9.

COF Evaluation for Multistory, Multibay Structures

To investigate COF values of multistory, multibay structures, the height-span ratio is taken as H/L = 4 m/8 m = 0.5. The mean strength of the beams on the top floor are taken to be 1,062 t cm, whereas those of other floors were all taken to be 2,124 t cm. The mean strength of the columns changed according to changes in COF. Structures from one to seven stories and from one to five bays were investigated. The computational target values for the tolerance level γ_0 = 0.8 are listed in Table 2 for β = 2 and in Table 3 for β = 3, where Columns 2–6 show the results of target COF values for each number of bays.

From Tables 2 and 3, one can see that, for a given number

TABLE 2. Target Values of COF for Multistory, Multibay Structures ($\beta = 2$, $\gamma = 0.8$)

Number of stories (1)	One bay (2)	Two bays (3)	Three bays (4)	Four bays (5)	Five bays (6)
1	1.32	1.74	2.36	3.32	5.07
2	1.57	2.35	3.42	5.05	7.90
3	1.79	2.83	4.27	6.25	9.35
4	1.97	3.21	4.82	7.03	10.36
5	2.12	3.51	5.30	7.68	11.07
6	2.23	3.74	5.67	8.17	11.58
7	2.41	3.98	5.96	8.55	11.86

TABLE 3. Target Values of COF for Multistory, Multibay Structures ($\beta = 3$, $\gamma = 0.8$)

Number of stories (1)	One bay (2)	Two bays (3)	Three bays (4)	Four bays (5)	Five bays (6)
1	1.23	1.49	1.83	2.28	2.92
2	1.41	1.91	2.57	3.42	4.53
3	1.59	2.33	3.22	4.42	5.91
4	1.75	2.68	3.82	5.25	7.06
5	1.91	3.01	4.35	6.03	8.22
6	2.29	3.29	4.82	6.66	9.06
7	2.81	3.56	5.19	7.20	9.83

TABLE 4. Target Values of COF for Multistory, Multibay Structures ($\beta = 2$, $\gamma = 0.9$)

Number of stories (1)	One bay (2)	Two bays (3)	Three bays (4)	Four bays (5)	Five bays (6)
1	1.20	1.29	1.48	1.67	1.92
2	1.24	1.53	1.86	2.24	2.67
3	1.36	1.73	2.17	2.67	3.20
4	1.52	1.90	2.44	3.05	3.70
5	1.69	2.03	2.64	3.33	4.07
6	1.94	2.14	2.80	3.54	4.34
7	2.19	2.26	2.95	3.73	4.60

TABLE 5. Target Values of COF for Multistory, Multibay Structures ($\beta = 3$, $\gamma = 0.9$)

Number of stories (1)	One bay (2)	Two bays (3)	Three bays (4)	Four bays (5)	Five bays (6)
1	1.10	1.20	1.33	1.45	1.58
2	1.18	1.38	1.60	1.87	2.12
3	1.26	1.56	1.86	2.20	2.60
4	1.34	1.70	2.10	2.55	3.06
5	1.40	1.85	2.32	2.86	3.39
6	1.56	1.97	2.50	3.13	3.79
7	1.69	2.08	2.73	3.39	4.09

of stories, the larger the number of bays, the larger will be the target value of COF. For a given number of bays, the larger the number of stories, the larger will be the target value of COF. One can also see that the target value of COF is more sensitive to the number of bays than to the number of stories.

The computational target values for the tolerance level $\gamma_0 = 0.9$ are listed in Table 4 for $\beta = 2$ and in Table 5 for $\beta = 3$, where Columns 2–6 show the results of target COF values for each number of bays. From Tables 4 and 5, one can see that target values for the tolerance level $\gamma_0 = 0.9$ are much smaller than those for the tolerance level $\gamma_0 = 0.8$.

CONCLUSIONS

A stochastic limit analysis procedure was developed, and the target value of COF that ensures frame structures collapse according to the beam-failure pattern was discussed in probabilistic terms. It was found that

- To ensure that the frame structures collapse according to the beam-failure pattern, a considerably larger value of COF is required in the case where both the loads and the member strengths are uncertain than in the case of a deterministic member strength.
- For a given number of stories, the larger the number of bays, the larger will be the target value of COF. Moreover, for a given number of bays, the larger the number of stories, the larger will be the target value of COF. The target value of COF is more sensitive to the number of bays than to the number of stories.
- The higher the reliability level is set when the structure is designed, the smaller the required target value of COF to limit the probability ratio to within a given tolerance level.
- The target value of COF is very sensitive to the tolerance level γ_0. For suitable stochastic evaluation of the target values of COF, it is therefore very important to set an appropriate tolerance level.

It should be noted that the investigation was conducted under some restrictive assumptions described in the paper. Some other factors that were not considered in the paper, such as second-order effects and axial deformation, type of ground motion and dynamic response, distribution type of random variables, definition of the beam-hinging pattern, and correlations among member strengths, may be important. To determine reasonable target COF with consideration of all of these factors requires further research.

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APPENDIX II. NOTATION

The following symbols are used in this paper:

$[E]$ = unit matrix;
$[H]$ = coefficient matrix for equilibrium equation;
M_{pb} = mean value of ultimate moment of beam;
M_{pc} = mean value of ultimate moment of column;
$\{P\}$ = load vector;
p_{f1} = occurrence probability of most likely failure mode;
p_{f2} = occurrence probability of second most likely failure mode;
$\{r\}$ = vector of member strength;
γ = probability ratio;
γ_0 = specific allowable level; and
λ = load factor.