

Estimating Load and Resistance Factors without Using Distributions of Random Variables

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Abstract

In this paper, a new method for estimating the load and resistance factors using the first three moments of random variables is proposed. Unlike the currently used method, the load and resistance factors can be determined without using distributions of random variables, and the present method needs neither the iterative computation of derivatives nor any design points. The present method can effectively reflect the characteristics of the skewness of random variables and the performance function, and generally provide much more accurate results than the second moment method. Thus, the present method should be convenient and more effective in estimating the load and resistance factors in practical engineering. Numerical examples are presented to demonstrate the advantages of the present method.

Keywords: load and resistance factors; third moment method; target mean resistance; simple formula

1. Introduction

As the insurance of the performance of a structure that must be accomplished under conditions of uncertainty, probabilistic analysis will be generally necessary for reliability-based structural design. However, a reliability-based structural design may also be developed without a complete probabilistic analysis. If the required safety factors are predetermined on the basis of specified probability-based requirements, a reliability-based design may be accomplished through the adoption of appropriate deterministic design criteria, e.g., the use of traditional safety factors.

For obvious reasons, design criteria should be as simple as possible; moreover, they should be developed in a form that is familiar to the users or designers. A practical format is the use of load amplification factors and resistance reduction factors, known as the LRFD format (Galambos *et al.*, 1982; Ellingwood *et al.*, 1982; Ang and Tang, 1984). That is, the nominal design loads are amplified by appropriate load factors and the nominal resistances are reduced by the corresponding resistance factors, and safety is assured if the factored resistance is at least equal to the factored loads. Load and resistance factors can be developed in order to obtain designs that achieve a prescribed level of reliability.

The load and resistance factors are generally determined using the first order reliability method

(FORM) (Hasofer and Lind, 1974; Rackwitz, 1976; Shinozuka, 1983), in which the design point should be determined and derivative-based iteration has to be used. Some simplified methods have been proposed in order to avoid iterative computation (Ugata, 2000; Mori, 2002; Mori and Maruyama, 2005). In almost all of the current methods, the basic random variables are assumed to have known cumulative distribution functions (CDFs) or probability density functions (PDFs). However, in reality, the CDF/PDFs of some of the basic random variables are often unknown due to the lack of statistical data. Therefore, it is important to find a way to obtain LRFD without using distributions of random variables.

The second moment (2M) method has also been used to determine the load and resistance factors, and these factors can be obtained even when the distributions of random variables are unknown. However, the 2M method is correct only when the performance function is a normal variable. For non-normal performance function, only the first two central moments of the performance function are apparently inadequate, and high-order moments will invariably be necessary.

In this paper, the basic principle of determining the load and resistance factors is reviewed and a new method for estimating the load and resistance factors using the first three moments of random variables is proposed. In order to determine the target mean resistance, a simple formula, which can avoid the iteration computation, is proposed. Since the only information used in the proposed method is the first three moments of random variables, the load and resistance factors can be determined even when the distributions of random variables are unknown. The accuracy of the simple formula is investigated and

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numerical examples are presented to demonstrate the advantages of the proposed method.

2. Third Moment Method for LRFD

The LRFD format is expressed as

$$\phi R_n \geq \sum \gamma_i S_{ni} \quad (1)$$

where ϕ = the resistance factor, γ_i = the partial load factor to be applied to load S_i , R_n = the nominal value of the resistance and S_{ni} = the nominal value of load S_i .

In reliability-based structural design, resistance factors ϕ and load factors γ_i should be determined in order to achieve a specified reliability. That is, the design format, Eq. 1, should be equivalent to the following design format in probability terms.

$$G(\mathbf{X}) = R - \sum S_i \quad (2)$$

$$P_f \leq P_{JT} \quad \text{or} \quad \beta \geq \beta_T \quad (3)$$

where R and S_i are the random variables representing the uncertainty included in resistance and load effects. P_f and β are the probability of failure and reliability index corresponding to the performance function Eq. 2. P_{JT} and β_T are the target probability of failure and target reliability index, respectively.

If R and S_i are mutually independent normal random variables, the second moment method is correct and the design formula becomes

$$\beta_{2M} \geq \beta_T \quad (4)$$

where

$$\beta_{2M} = \frac{\mu_G}{\sigma_G} = \frac{\mu_R - \sum \mu_{Si}}{\sigma_G} \quad (5a)$$

$$\sigma_G = \sqrt{\sigma_R^2 + \sum \sigma_{Si}^2} \quad (5b)$$

where β_{2M} is the 2M reliability index, μ_R and μ_{Si} are the mean values of R and S_i , σ_R and σ_{Si} are the standard deviations of R and S_i .

Substituting Eq. 5 in Eq. 4 produces

$$\mu_R (1 - \alpha_R V_R \beta_T) \geq \sum \mu_{Si} (1 + \alpha_{Si} V_{Si} \beta_T) \quad (6)$$

Comparing Eq. 6 with Eq. 1, the load and resistance factors may be expressed as

$$\phi = (1 - \alpha_R V_R \beta_T) \frac{\mu_R}{R_n}, \quad \gamma_i = (1 + \alpha_{Si} V_{Si} \beta_T) \frac{\mu_{Si}}{S_{ni}} \quad (7)$$

where V_R and V_{Si} are the coefficients of variation, respectively, of R and S_i , and α_R and α_{Si} are the separating factors, respectively, for R and S_i ,

$$\alpha_R = \frac{\sigma_R}{\sigma_G}, \quad \alpha_{Si} = \frac{\sigma_{Si}}{\sigma_G} \quad (8)$$

When R and S_i are non-normal random variables, the reliability index expressed in Eq. 5 is not correct, and the first two moments are inadequate, so higher-order moments will be invariably necessary.

In the present study, the third moment (3M) reliability index is used as the reliability index for the performance function of Eq. 2. Substituting the 3M

reliability index in the design format described in Eq. 3 produces,

$$\beta_{3M} \geq \beta_T \quad (9)$$

where the 3M reliability index β_{3M} is expressed as (Zhao *et al.*, 2006)

$$\beta_{3M} = \frac{1}{\alpha_{3G}} \left(3 - \sqrt{9 + \alpha_{3G}^2 - 6\alpha_{3G}\beta_{2M}} \right) \quad (10)$$

where α_{3G} is the skewness of $G(\mathbf{X})$. The α_{3G} of Eq. 2 can be computed by

$$\alpha_{3G} = \frac{1}{\sigma_G^3} (\alpha_{3R} \sigma_R^3 - \sum \alpha_{3Si} \sigma_{Si}^3) \quad (11)$$

where α_{3R} and α_{3Si} are the skewness of R and S_i .

The applicable range of the 3M method is given as (Zhao *et al.*, 2006).

$$-\frac{120r}{\beta_{2M}} \leq \alpha_{3G} \leq \frac{40r}{\beta_{2M}}, \quad |\alpha_{3G}| \leq 1 \quad (12a)$$

where r is the relative difference among several 3M reliability indices, for example, if $r=2\%$, then

$$-\frac{2.4}{\beta_{2M}} \leq \alpha_{3G} \leq \frac{0.8}{\beta_{2M}}, \quad |\alpha_{3G}| \leq 1 \quad (12b)$$

Substituting Eq. 10 for Eq. 9, one obtains

$$\beta_{2M} \geq \beta_T - \frac{1}{6} \alpha_{3G} (\beta_T^2 - 1) \quad (13)$$

Denoting the right side of Eq. 13 as β_{2T} , one obtains

$$\beta_{2M} \geq \beta_{2T} \quad (14)$$

$$\beta_{2T} = \beta_T - \frac{1}{6} \alpha_{3G} (\beta_T^2 - 1) \quad (15)$$

Eq. 14 is the same as Eq. 9. This means that if β_{2M} is at least equal to β_{2T} , β_{3M} will be at least equal to β_T , and the reliability-based design conditions will be satisfied. Therefore, β_{2T} can be considered a target value of β_{2M} . Hereafter, β_{2T} is called the target 2M reliability index.

Since Eq. 14 is as the same as Eq. 4 except that the right side is β_{2T} , the load and resistance factors corresponding to Eq. 14 can be easily obtained by substituting β_T in the right side of Eq. 6 with β_{2T} . The design formula becomes

$$\mu_R (1 - \alpha_R V_R \beta_{2T}) \geq \sum \mu_{Si} (1 + \alpha_{Si} V_{Si} \beta_{2T}) \quad (16)$$

And the load and resistance factors are obtained as

$$\phi = (1 - \alpha_R V_R \beta_{2T}) \frac{\mu_R}{R_n} \quad (17a)$$

$$\gamma_i = (1 + \alpha_{Si} V_{Si} \beta_{2T}) \frac{\mu_{Si}}{S_{ni}} \quad (17b)$$

3. Estimation of the Mean Value of Resistance

3.1 The Iteration Method

Since the load and resistance factors are determined when the reliability index is equal to the target reliability index, the mean value of resistance should be determined under this condition (hereafter referred

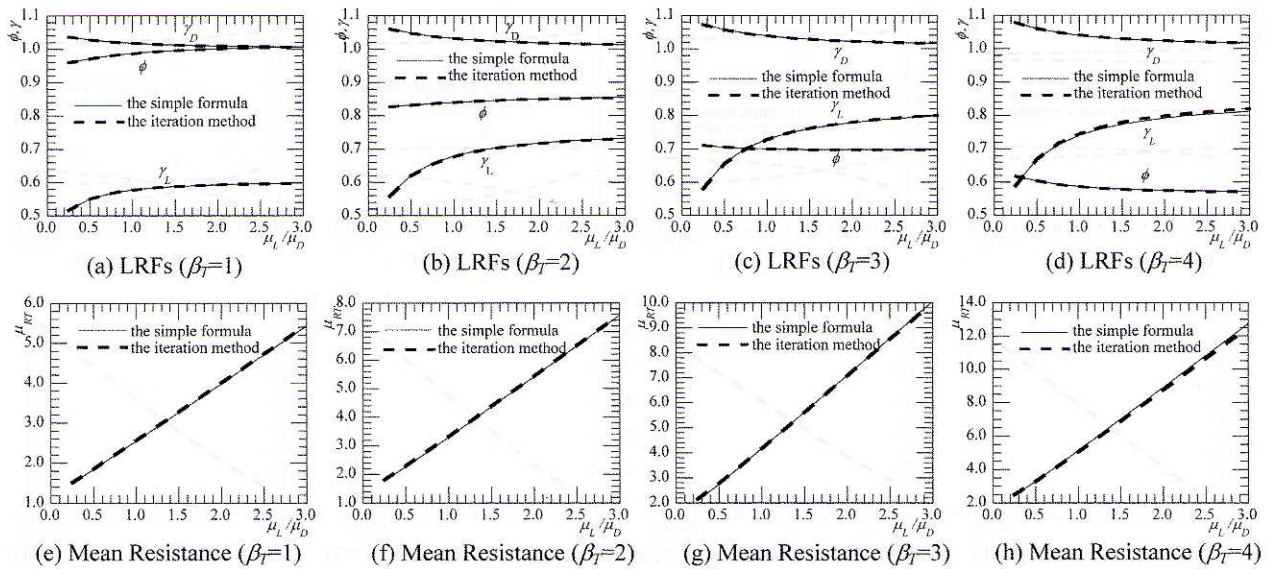


Fig.1. Figure for Case 1: $G(X)=R-(D+L)$

to as the target mean resistance). The target mean resistance is computed using the following equation (Takada, 2001).

$$\mu_{Rk} = \mu_{Rk-1} + (\beta_T - \beta_{k-1})\sigma_G \quad (18)$$

where μ_{Rk} and μ_{Rk-1} are the k th and $(k-1)$ th iteration values of the mean value of resistance; β_{k-1} is the $(k-1)$ th iteration value of the third moment reliability index.

The procedures for determining the load and resistance factors using Eq. 18 are as follows:

- (1) Assume $\mu_{R0} = \sum \mu_{Si}$.
- (2) Calculate μ_G , σ_G , and α_{3G} using Eq. 5 and Eq. 11, and determine β_{2M} with the aid of Eq. 5.
- (3) Calculate β_{3M} using Eq. 10.
- (4) Calculate μ_{Rk} using Eq. 18.
- (5) Repeat the computation processes of 2-4 until $|\beta_T - \beta_{k-1}| < 0.0001$, and then the target mean resistance is determined.
- (6) Calculate σ_G , α_{3G} , and β_{2T} using Eq. 5, Eq. 11, and Eq. 15, respectively, and calculate α_R and α_{Si} with the aid of Eq. 8.
- (7) Determine the load and resistance factors using Eq. 17.

3.2 A Simple Formula for Approximating the Iteration Computation

The determination of the target mean resistance using the above method will require an iteration computation, but this is inconvenient for users or designers. For obvious reasons, the computation should be as simple and accurate as possible for users and designers. In the following, a simple formula for approximating the iteration computation is proposed.

At the limit state, according to Eq. 5 and Eq. 14, one obtains

$$\mu_R = \sum \mu_{Si} + \beta_{2T} \sigma_G \quad (19)$$

For the above equation, μ_{Si} is known, and since μ_R remains to be determined, the values of σ_G and β_{2T} ,

which are functions of μ_R , are still unknown. Thus, in order to obtain the target mean resistance, an initial value of the mean resistance μ_{R0} has to be assumed.

Note, Eq. 19 can be expressed as

$$\mu_R = \sum \mu_{Si} + \beta_{2T} \sigma_G = \sum \mu_{Si} + \left[\beta_T - \frac{1}{6} \alpha_{3G} (\beta_T^2 - 1) \right] \times \left(\sqrt{1 + \frac{(\mu_R V_R)^2}{\sum \sigma_{Si}^2}} \right) \sqrt{\sum \sigma_{Si}^2} \quad (20)$$

In the applicable range of the 3M method (Zhao *et al.*, 2006), the following approximation was obtained through trial and error.

$$\left[\beta_T - \frac{1}{6} \alpha_{3G} (\beta_T^2 - 1) \right] \times \left(\sqrt{1 + \frac{(\mu_R V_R)^2}{\sum \sigma_{Si}^2}} \right) \approx \sqrt{\beta_T^3} \quad (21)$$

Thus, the initial value μ_{R0} can be assumed to be

$$\mu_{R0} = \sum \mu_{Si} + \sqrt{\beta_T^3 \sum \sigma_{Si}^2} \quad (22)$$

Based on the discussion above, a simple formula for approximating the iteration computation of the target mean resistance is proposed as

$$\mu_{RT} = \sum \mu_{Si} + \beta_{2T0} \sigma_{G0} \quad (23)$$

where μ_{RT} = the target mean resistance, σ_{G0} = the standard deviation of $G(X)$, and β_{2T0} = the target 2M reliability index. σ_{G0} and β_{2T0} are obtained using μ_{R0} .

The procedures for determining the load and resistance factors using the present simple formula are as follows:

- (1) Calculate μ_{R0} using Eq. 22.
- (2) Calculate σ_{G0} , α_{3G0} , and β_{2T0} using Eq. 5, Eq. 11, and Eq. 15, respectively, and determine μ_{RT} with the aid of Eq. 23.
- (3) Calculate σ_G , α_{3G} , and β_{2T} using Eq. 5, Eq. 11, and Eq. 15, respectively, and calculate σ_R and α_{Si} with the aid of Eq. 8.
- (4) Determine the load and resistance factors using Eq. 17.

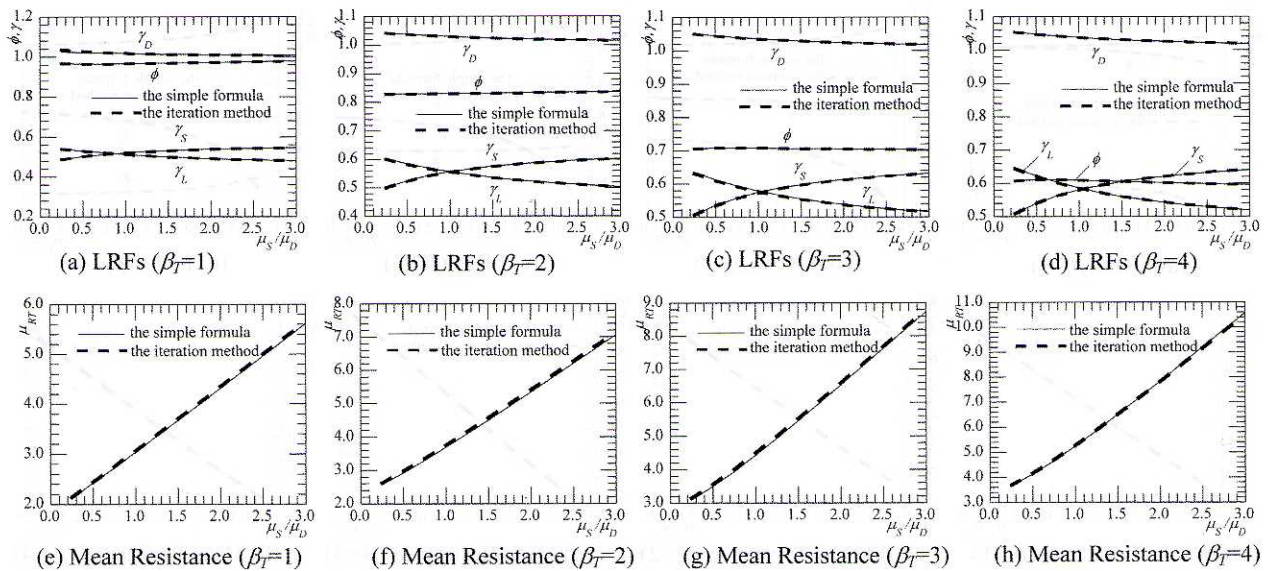


Fig.2. Figure for Case 2: $G(\mathbf{X})=R-(D+L+S)$

3.3 Accuracy of the Simple Formula

In order to investigate the efficiency of the proposed simple formula, several cases are examined under different conditions.

Case 1: Consider the following performance function where

$$G(\mathbf{X}) = R - (D + L) \quad (24)$$

R = resistance, with unknown CDF, $\mu_R/R_n = 1.1$, $V = 0.15$, $\alpha_{3R} = 0.453$,

D = dead load, with unknown CDF, $\mu_D/D_n = 1.0$, $V = 0.1$, $\alpha_{3D} = 0.0$, and

L = live load, with unknown CDF, $\mu_L/L_n = 0.45$, $V = 0.4$, $\alpha_{3L} = 1.264$.

Consider the mean value of D with $\mu_D = 1.0$ and the target reliability index $\beta_T = 1.0, 2.0, 3.0$, and 4.0 . Determine the target mean resistance and the load and resistances factors for any given μ_L .

The load and resistance factors obtained using the simple formula are illustrated in Figs.1.(a)-(d), compared with the corresponding factors obtained using the iteration calculation of the 3M method for $\beta_T = 1.0, 2.0, 3.0$, and 4.0 . The target mean resistances obtained using the simple formula and those obtained using the iteration calculation are illustrated in Figs.1.(e)-(h) for $\beta_T = 1.0, 2.0, 3.0$, and 4.0 . From Fig.1., one can see that the load and resistance factors and the target mean resistances obtained by the simple formula and the iteration calculation are essentially the same for a given target reliability index. Clearly, for this performance function, the simple formula approximates the iteration calculation method quite well.

Case 2: Consider the following performance function

$$G(\mathbf{X}) = R - (D + L + S) \quad (25)$$

where

R = resistance, with unknown CDF, $\mu_R/R_n = 1.1$, $V =$

0.15 , $\alpha_{3R} = 0.453$,

D = dead load, with unknown CDF, $\mu_D/D_n = 1$, $V = 0.1$, $\alpha_{3D} = 0.0$,

L = live load, with unknown CDF, $\mu_L/L_n = 0.45$, $V = 0.4$, $\alpha_{3L} = 1.264$, and

S = snow load, with unknown CDF, $\mu_S/S_n = 0.47$, $V = 0.25$, $\alpha_{3S} = 1.140$.

Consider the mean value of D, L with $\mu_D = 1.0$, $\mu_L/\mu_D = 0.5$, and the target reliability index $\beta_T = 1.0, 2.0, 3.0$, and 4.0 . Determine the target mean resistance and the load and resistance factors for any given μ_S .

The load and resistance factors obtained using the simple formula are illustrated in Figs.2.(a)-(d), compared with the corresponding factors obtained using the iteration calculation of the 3M method for $\beta_T = 1.0, 2.0, 3.0$, and 4.0 . The target mean resistances obtained using the simple formula and those obtained using the iteration calculation are illustrated in Figs.2.(e)-(h) for $\beta_T = 1.0, 2.0, 3.0$, and 4.0 . One can see from Fig.2. that the load and resistance factors and the target mean resistances obtained by the simple formula and the iteration calculation are essentially the same for a given target reliability index. That is to say, the iteration calculation can be replaced by the simple formula for this performance function.

Case 3: Consider the following performance function

$$G(\mathbf{X}) = R - (D + L + E) \quad (26)$$

where

R = resistance, with unknown CDF, $\mu_R/R_n = 1.1$, $V = 0.3$, $\alpha_{3R} = 0.927$,

D = dead load, with unknown CDF, $\mu_D/D_n = 1$, $V = 0.1$, $\alpha_{3D} = 0.0$,

L = live load, with unknown CDF, $\mu_L/L_n = 0.45$, $V = 0.4$, $\alpha_{3L} = 1.264$, and

E = earthquake load, with unknown CDF, $\mu_E/E_n = 0.16$, $V = 1.3$, $\alpha_{3E} = 6.097$.

Consider the mean value of D, L with $\mu_D = 1.0$, $\mu_L/$

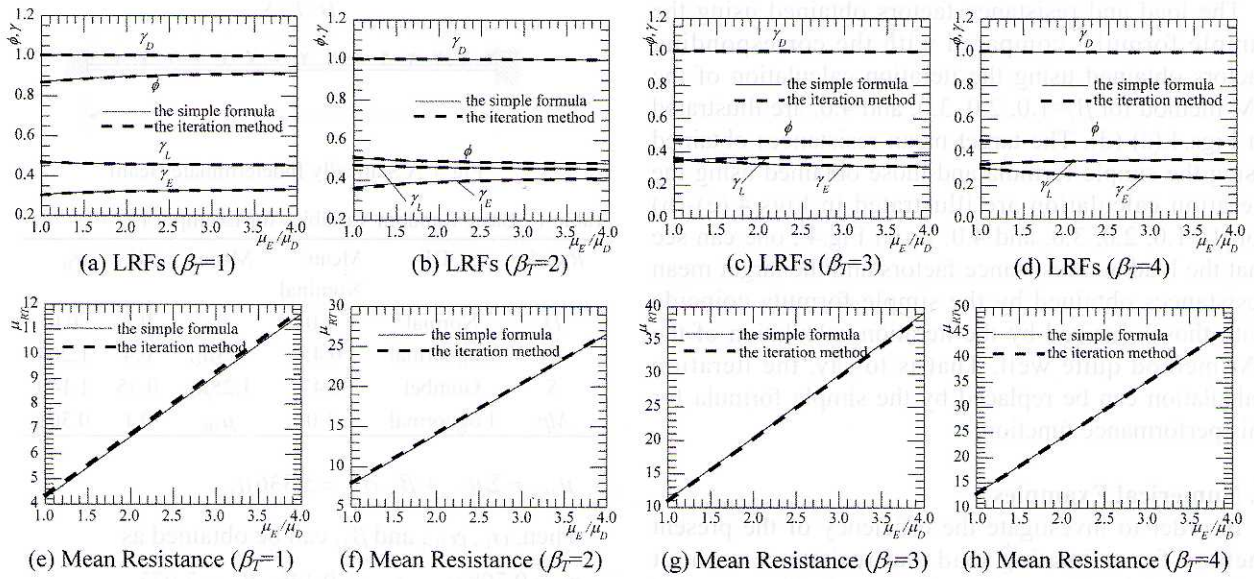


Fig.3. Figure for Case 3: $G(X)=R-(D+L+E)$

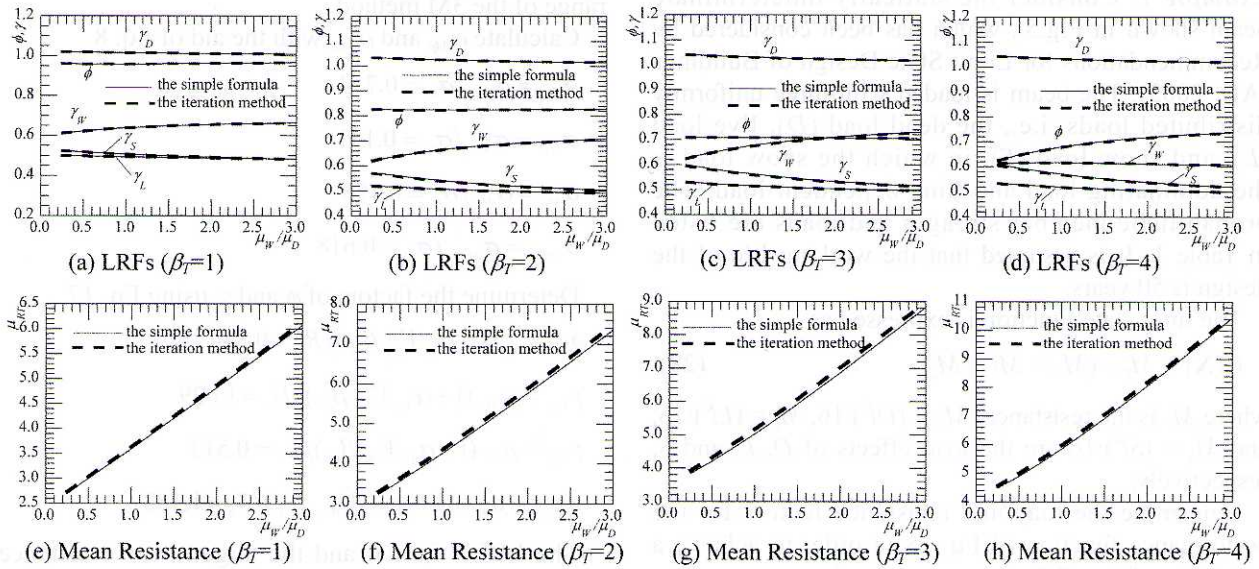


Fig.4. Figure for Case 4: $G(X)=R-(D+L+S+W)$

$\mu_D = 0.5$, and the target reliability index $\beta_T = 1.0, 2.0, 3.0$, and 4.0 . Determine the target mean resistance and the load and resistance factors for any given μ_E .

The load and resistance factors obtained using the simple formula, compared with the corresponding factors obtained using the iteration calculation for $\beta_T=1.0, 2.0, 3.0$, and 4.0 , are illustrated in Figs.3.(a)-(d). The target mean resistances obtained using the simple formula and those obtained using the iteration calculation are illustrated in Figs.3.(e)-(h) for $\beta_T=1.0, 2.0, 3.0$, and 4.0 . From Fig.3., one can see that the load and resistance factors and the target mean resistances obtained by the simple formula and the iteration computation are almost the same. Clearly, for this performance function, the simple formula approximates the iteration calculation method quite well.

Case 4: Consider the following performance function

$$G(X) = R - (D + L + S + W) \quad (27)$$

where

R = resistance, with unknown CDF, $\mu_R/R_n = 1.1$, $V = 0.15$, $\alpha_{3R} = 0.453$,

D = dead load, with unknown CDF, $\mu_D/D_n = 1$, $V = 0.1$, $\alpha_{3D} = 0.0$,

L = live load, with unknown CDF, $\mu_L/L_n = 0.45$, $V = 0.4$, $\alpha_{3L} = 1.264$,

S = snow load, with unknown CDF, $\mu_S/S_n = 0.47$, $V = 0.25$, $\alpha_{3S} = 1.140$, and

W = wind load, with unknown CDF, with $\mu_W/W_n = 0.6$, $V = 0.2$, $\alpha_{3W} = 1.140$.

Consider the mean value of D, L, S with $\mu_D = 1.0$, $\mu_L/\mu_D = 0.5$, $\mu_S/\mu_D = 0.5$, and the target reliability index $\beta_T = 1.0, 2.0, 3.0$, and 4.0 . Determine the target mean resistance and the corresponding load and resistance factors for any given μ_W .

The load and resistance factors obtained using the simple formula, compared with the corresponding factors obtained using the iteration calculation of the 3M method for $\beta_T=1.0, 2.0, 3.0$, and 4.0 , are illustrated in Figs.4.(a)-(d). The target mean resistances obtained using the simple formula and those obtained using the iteration calculation are illustrated in Figs.4.(e)-(h) for $\beta_T=1.0, 2.0, 3.0$, and 4.0 . From Fig.4., one can see that the load and resistance factors and the target mean resistances obtained by the simple formula coincide with those obtained by the iteration calculation of the 3M method quite well. That is to say, the iteration calculation can be replaced by the simple formula for this performance function.

4. Numerical Examples

In order to investigate the efficiency of the present method for estimating load and resistance without using distributions of random variables, several examples are examined under different conditions.

Example 1: Consider the statically indeterminate beam shown in Fig.5., which has been considered by Recommendations for Limit State Design of Buildings (AIJ, 2002). The beam is loaded with three uniformly distributed loads, i.e., the dead load (D), live load (L), and snow load (S), in which the snow load is the dominating load and time-dependent load. The probabilistic member strength and loads are listed in Table 1. It is assumed that the working life of the design is 50 years.

The limit state function is expressed as

$$G(\mathbf{X}) = M_p - (M_D + M_L + M_S) \quad (28)$$

where M_p is the resistance; $M_D = (Dl^2)/16$, $M_L = (Ll^2)/16$, and $M_S = (Sl^2)/16$ are the load effects of D , L , and S , respectively.

Determine the load and resistance factors for the performance function of Eq. 28 in order to achieve a reliability of $\beta_T = 2.0$.

Because S is a Gumbel random variable, the probability distribution of the maximum S over 50 years is also the Gumbel distribution (Melchers, 1999). The values of mean, mean/nominal, coefficient of variation, and skewness corresponding to the maximum snow load over 50 years are readily obtained as: $\mu_{S50} = 2.585\mu_D$, $\mu_{S50}/S_n = 0.972$, $V_{S50} = 0.169$, and $\alpha_{3S50} = 1.140$.

According to Eq. 22

$$\mu_{Mp0} = \Sigma \mu_{M_{Si}} + \sqrt{\beta_T^3 \Sigma \sigma_{M_{Si}}^2} = 5.199\mu_{MD}, \quad \mu_{MD} = (\mu_D l^2)/16$$

σ_{G0} , α_{3G0} , and β_{2T0} can be obtained using Eq. 5, Eq. 11, and Eq. 15, respectively,

$$\sigma_{G0} = \sqrt{\sigma_{M_{p0}}^2 + \Sigma \sigma_{M_{Si}}^2} = 0.697\mu_{MD}$$

$$\alpha_{3G0} = (\alpha_{3M_p} \sigma_{M_{p0}}^3 - \Sigma \alpha_{3M_{Si}} \sigma_{M_{Si}}^3) / \sigma_{G0}^3 = -0.163$$

$$\beta_{2T0} = \beta_T - \alpha_{3G0} (\beta_T^3 - 1) / 6 = 2.082$$

The target mean resistance μ_{MpT} can be estimated with the aid of Eq. 23.

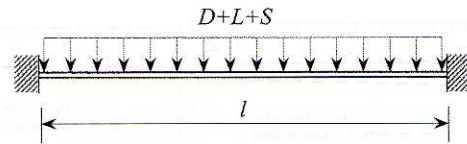


Fig.5. A Statically Indeterminate Beam

Table 1. Basic Random Variables for Example 1

R or S_i	CDF	Mean/ Nominal	Mean	V_i	α_{3i}
D	Normal	1.0	μ_D	0.1	0.0
L	Lognormal	0.45	$0.3\mu_D$	0.4	1.264
S	Gumbel	0.47	$1.25\mu_D$	0.35	1.140
M_p	Lognormal	1.0	μ_{Mp}	0.1	0.301

$$\mu_{MpT} = \Sigma \mu_{M_{Si}} + \beta_{2T0} \sigma_{G0} = 5.336\mu_{MD}$$

Then, σ_G , α_{3G} , and β_{2T} can be obtained as

$$\sigma_G = 0.708\mu_{MD}, \quad \alpha_{3G} = -0.146, \quad \beta_{2T} = 2.073$$

Since $-1.0 < \alpha_{3G} = -0.146 < 0.386$, it is in the applicable range of the 3M method.

Calculate α_{Mp} and α_{MSi} with the aid of Eq. 8

$$\alpha_{Mp} = \sigma_{Mp} / \sigma_G = 0.754$$

$$\alpha_{MD} = \sigma_{MD} / \sigma_G = 0.141$$

$$\alpha_{ML} = \sigma_{ML} / \sigma_G = 0.170$$

$$\alpha_{MS50} = \sigma_{MS50} / \sigma_G = 0.618$$

Determine the factors of ϕ and γ_i using Eq. 17

$$\phi = \mu_{Mp} (1 - \alpha_{Mp} V_{Mp} \beta_{2T}) / R_n = 0.844$$

$$\gamma_{MD} = \mu_{MD} (1 + \alpha_{MD} V_{MD} \beta_{2T}) / D_n = 1.029$$

$$\gamma_{ML} = \mu_{ML} (1 + \alpha_{ML} V_{ML} \beta_{2T}) / L_n = 0.513$$

$$\gamma_{MS50} = \mu_{MS50} (1 + \alpha_{MS50} V_{MS50} \beta_{2T}) / S_n = 1.183$$

The LRFD format and the target mean resistances using the present method are obtained as

$$0.84M_{pn} \geq 1.03M_{Dn} + 0.51M_{Ln} + 1.18M_{Sn}$$

$$\mu_{Mp} \geq 5.37\mu_{MD}$$

where $M_{Dn} = (D_n l^2)/16$, $M_{Ln} = (L_n l^2)/16$, and $M_{Sn} = (S_n l^2)/16$.

The LRFD format and the target mean resistances using the 2M method are obtained as

$$0.83M_{pn} \geq 1.02M_{Dn} + 0.50M_{Ln} + 1.14M_{Sn}$$

$$\mu_{Mp} \geq 5.28\mu_{MD}$$

The LRFD format and the target mean resistances using FORM are obtained as

$$0.89M_{pn} \geq 1.02M_{Dn} + 0.46M_{Ln} + 1.28M_{Sn}$$

$$\mu_{Mp} \geq 5.32\mu_{MD}$$

The LRFD format and the target mean resistances using the practical method (Mori, 2002; AIJ, 2002) are

Table 2. Target Mean Resistances Obtained by Different Methods

	Present	2M	FORM
$V_{Mp}=0.05$	$5.14\mu_{MD}$	$4.96\mu_{MD}$	$5.05\mu_{MD}$
$V_{Mp}=0.1$	$5.37\mu_{MD}$	$5.28\mu_{MD}$	$5.32\mu_{MD}$
$V_{Mp}=0.2$	$6.23\mu_{MD}$	$6.73\mu_{MD}$	$6.18\mu_{MD}$
$V_{Mp}=0.3$	$7.50\mu_{MD}$	$9.80\mu_{MD}$	$7.47\mu_{MD}$

$$0.88M_{Pn} \geq 1.02M_{Dn} + 0.49M_{Ln} + 1.28M_{Sn}$$

$$\mu_{Mp} \geq 5.40\mu_{MD}$$

From this example, one can see that although the load and resistance factors obtained using the present method are different from those obtained using FORM, the design resistances obtained by the present method are quite close to those of FORM. For this case, the 2M method also provides good results.

In order to demonstrate the superiority of the present method over the 2M method, the coefficients of variation of the resistance are assumed to be 0.05, 0.1, 0.2, and 0.3, respectively, and all the others are the same as the original values. The target mean resistances varying with the coefficients of variation of the resistance obtained by the present method are listed in Table 2, together with those obtained by the 2M method and FORM. From this table, one can see that the results of the present method are close to those obtained by FORM, and generally the present method provides much more accurate results than the 2M method.

Example 2: The second example considers the following performance function

$$G(\mathbf{X}) = R - (D + L + E) \quad (29)$$

where R is the resistance; D denotes the dead load effect; L denotes the live load effect, and E is the maximum earthquake load effect over 50 years.

The probabilistic information of R , D , L , and E is listed in Table 3. Determine the load and resistance factors for the performance function of Eq. 29, in order to achieve a reliability of $\beta_T=2.4$.

Similarly, the LRFD format and the target mean resistances using the present method are obtained as

$$0.39R_n \geq 1.0D_n + 0.46L_n + 0.4E_n$$

$$\mu_R \geq 35.69\mu_D$$

The LRFD format and the target mean resistances using the 2M method are obtained as

$$0.41R_n \geq 1.0D_n + 0.46L_n + 0.4E_n$$

$$\mu_R \geq 33.95\mu_D$$

The present method can effectively reflect the characteristics of the skewness of random variables and the performance function. For illustration, the skewness of R is assumed to be 0.6, and the other provisions are the same. Because this change will result in variation of the skewness of the performance function, the LRFD format and the target mean resistances using the

Table 3. Basic Random Variables for Example 2

R or S_i	CDFs	μ_{Si}/S_{mi} or μ_R/R_n	Mean	V_i	α_{3i}
R	unknown	1.10	μ_R	0.3	0.927
D	unknown	1.0	μ_D	0.1	0.0
L	unknown	0.45	$0.5\mu_D$	0.4	1.264
E	unknown	0.16	$5\mu_D$	1.3	6.097

present method are obtained as

$$0.35R_n \geq 1.0D_n + 0.46L_n + 0.41E_n$$

$$\mu_R \geq 40.66\mu_D$$

In contrast, the results obtained by using the 2M method remain the same, and apparently, the 2M method cannot flexibly reflect the variation of the skewness of random variables and the performance function.

Example 3: Consider the following nonlinear performance function of the fully plastic flexural capacity of a steel beam section

$$G(\mathbf{X}) = YZ - M \quad (30)$$

where

Y = the yield strength of steel, a lognormal variable.

Z = section modulus of the section, a lognormal variable.

M = the applied bending moment at the pertinent section, a Gumbel variable.

Determine the mean design section modulus for the performance function of Eq. 30, in order to achieve a reliability of $\beta_T=2.5$.

The purpose of this design problem is to determine the appropriate μ_Z for any given μ_M to satisfy the required reliability. The mean value of Y is $\mu_Y = 276\text{Mpa}$ and the coefficients of variation of Y , Z , and M are $V_Y = 0.1$, $V_Z = 0.05$, and $V_M = 0.3$, respectively. We determine the required design section as follows.

First, to calculate the value of μ_{Z0}

$$\mu_G = \mu_Y \mu_{Z0} - \mu_M = \sqrt{\beta_T^2 \sigma_M^2}$$

$$\mu_{Z0} = \left(\sqrt{\beta_T^2 \sigma_M^2} + \mu_M \right) / \mu_Y = 7.920 \times 10^{-3} \mu_M$$

Let $R = YZ$, then

$$\begin{aligned} \sigma_{R0} &= \sigma_{YZ0} = \sqrt{(\mu_Y \mu_{Z0})^2 [(1+V_Y^2)(1+V_Z^2) - 1]} \\ &= 0.2446\mu_M \end{aligned}$$

Therefore

$$\sigma_{G0} = \sqrt{\sigma_{R0}^2 + \sigma_M^2} = 0.3871\mu_M$$

The skewnesses of Y , Z , and M are readily obtained as

$$\alpha_{3Y} = 0.301, \quad \alpha_{3Z} = 0.150, \quad \alpha_{3M} = 1.14$$

The skewnesses of R can be obtained by

$$\begin{aligned} \alpha_{3R0} &= \alpha_{3YZ0} = [(\alpha_{3Y} V_Y^3 + 3V_Y^2 + 1)(\alpha_{3Z} V_Z^3 + 3V_Z^2 + 1) \\ &\quad - 3(V_Y^2 + 1)(V_Z^2 + 1) + 2] / V_{YZ0}^3 = 0.3371 \end{aligned}$$

Thus

$$\alpha_{3G_0} = [(\alpha_{3R_0} \sigma_{R_0}^3 - \alpha_{3M} \sigma_M^3) / \sigma_{G_0}^3] = -0.4456$$

According to Eq. 12, it is in the applicable range of the 3M method.

At the limit state, the appropriate μ_{ZT} is obtained as

$$\mu_{ZT} = \left\{ \mu_M + \left[\beta_T - \frac{1}{6} a_{3G_0} (\beta_T^2 - 1) \right] \sigma_{G_0} \right\} / \mu_Y = 0.0077 \mu_M$$

At the limit state, the design result of μ_{ZT} using FORM is obtained as $\mu_{ZT} = 0.0079 \mu_M$. The result obtained by the 2M method is $\mu_{ZT} = 0.0070 \mu_M$. One can see that the result of the present method is in close agreement with that obtained by FORM, while the 2M method provides the wrong result.

From the numerical examples, one can see that the present method needs neither the iterative computation of derivatives nor any design points. The designers or users can easily produce a reliability-based design with the aid of the present method. Apparently, if the first three moments of the basic random variables are known, the reliability-based design can be realized using the present method, even when the probability distributions of the basic random variables are unknown.

5. Conclusions

A new method for estimating the load and resistance factors using the first three moments of random variables is proposed. It is found that

- (1) The present expressions of load and resistance factors are simple and explicit.
- (2) Since the present method is only related to the mean, standard deviation, and skewness of each variable, the load and resistance factors can be determined without using distributions of random variables.
- (3) Although the proposed formula is quite simple, it is accurate enough to replace the iteration computation of the third moment method.
- (4) Although the load and resistance factors obtained by the present method are very different from those obtained from FORM, the target mean resistances obtained by both the methods are almost the same.
- (5) Derivative-based iteration and the design point, which are necessary in FORM, are not required in the proposed methods. For this reason, the proposed method is simpler to apply.
- (6) The present method can effectively reflect the characteristics of the skewness of random variables and the performance function, and generally provide much more accurate results than the second-moment method.

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