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4P-LAMBDA DISTRIBUTION AND ITS APPLICATIONS TO STRUCTURAL RELIABILITY ASSESSMENT

4P-Lambda分布及びその構造信頼性評価への応用

Yan-Gang ZHAO, Zhao-Hui LU** and Tetsuro ONO****
趙 衍 剛, 盧 朝 輝, 小野徹郎

In this paper, the four-parameter (4P) Lambda distribution, whose four parameters can be easily determined using the published tables in terms of the mean value, standard deviation, skewness, and kurtosis of the sample data, is investigated. From the investigation of this paper, one can see that this distribution, having characteristics of simplicity, generality, and flexibility, can be applied as a candidate distribution in fitting statistical data of basic random variables and can be used to represent or approximate the most popular one-, two-, and three-parameter distributions. A fourth-moment reliability index based on this distribution is derived and its application to structural reliability assessment is discussed. Numerical examples are presented to demonstrate these advantages.

Keywords: *structural reliability, probability distributions, statistical moments, data fitting, fourth-moment reliability index*
構造信頼性, 確率分布, 統計モーメント, データフィット, 4次モーメント信頼性指標

1. INTRODUCTION

In structural reliability evaluation, the basic random variables representing uncertain quantities, such as loads, environmental factors, material properties, structural dimensions, and variables introduced to account for modeling and prediction errors, are assumed to have known cumulative distribution functions (CDFs) or probability density functions (PDFs). Determination of the probability distributions of these basic random variables is essential for the accurate evaluation of the reliability of a structure.

Many methods for determining the probability distributions have been developed, such as Bayesian approach[1], B-spline function[2], theoretical approach[3], and others. Usually, the basic method for determining the required distribution is to fit the histogram of the statistical data of a variable with a candidate distribution[4], and apply statistical goodness-of-fit tests. Generally, such candidate distribution would have parameters that may be evaluated from the mean value and standard deviation of the statistical data. It has been reported[5] that the two-parameter (2P) distributions may not be appropriate when the skewness of the statistical data is important and must be reflected in the distribution. Thus, the three-parameter (3P) distributions[5], which can effectively reflect the information of skewness as well as the mean value and standard deviation of statistical data, have been suggested as the candidate distribution. However, the 3P distributions may be not flexible enough to reflect the kurtosis of statistical data of a random variable, and distributions that can be determined by effectively using the information of kurtosis as well as the mean value, standard deviation, and skewness of the statistical data are required.

On the other hand, in structural reliability analysis, for a performance function $z=G(\mathbf{X})$, if the first four moments of $G(\mathbf{X})$ are obtained, the probability of failure P_f can be estimated from the relationship between the CDF of $G(\mathbf{X})$ and its first four central moments. Therefore, it is convenient to have a distribution whose parameters are determined by its first four moments.

In this paper, the four-parameter Lambda (4P-Lambda) distribution[6], whose four parameters can be easily determined using the published tables[7] in terms of the mean value, standard deviation, skewness, and kurtosis of the sample data, is investigated. From the investigation of this paper, one can see that this distribution, having characteristics of simplicity, generality, and flexibility, can be applied as a candidate distribution in fitting statistical data of basic random variables and can be used to represent or approximate the most popular one-, two-, and three-parameter distributions. A fourth-moment reliability index based on this distribution is derived and its application to structural reliability assessment is discussed.

2. THE 4P-LAMBDA DISTRIBUTION

2.1 Definition of the Distribution and Moments

The 4P-Lambda distribution is defined by its inverse cumulative distribution function[6]

$$z = R(p) = \lambda_1 + \left[p^{\lambda_3} - (1-p)^{\lambda_4} \right] / \lambda_2, \quad (0 \le p \le 1) \tag{1}$$

in which λ_1, λ_2, λ_3, and λ_4 are the four parameters of the distribution, λ_1 is a location parameter, λ_2 is a scale parameter, and λ_3 and λ_4 are shape parameters.

The probability density function corresponding to Eq. 1 is given by

* Assoc. Prof., Dept. of Architecture, Nagoya Institute of Technology, Dr. Eng. 名古屋工業大学大学院社会工学専攻 助教授・工博
** Graduate Student, Dept. of Architecture, Nagoya Institute of Technology, M. Eng. 名古屋工業大学大学院社会工学専攻 大学院生・工修
*** Prof., Dept. of Architecture, Nagoya Institute of Technology, Dr. Eng. 名古屋工業大学大学院社会工学専攻 教授・工博

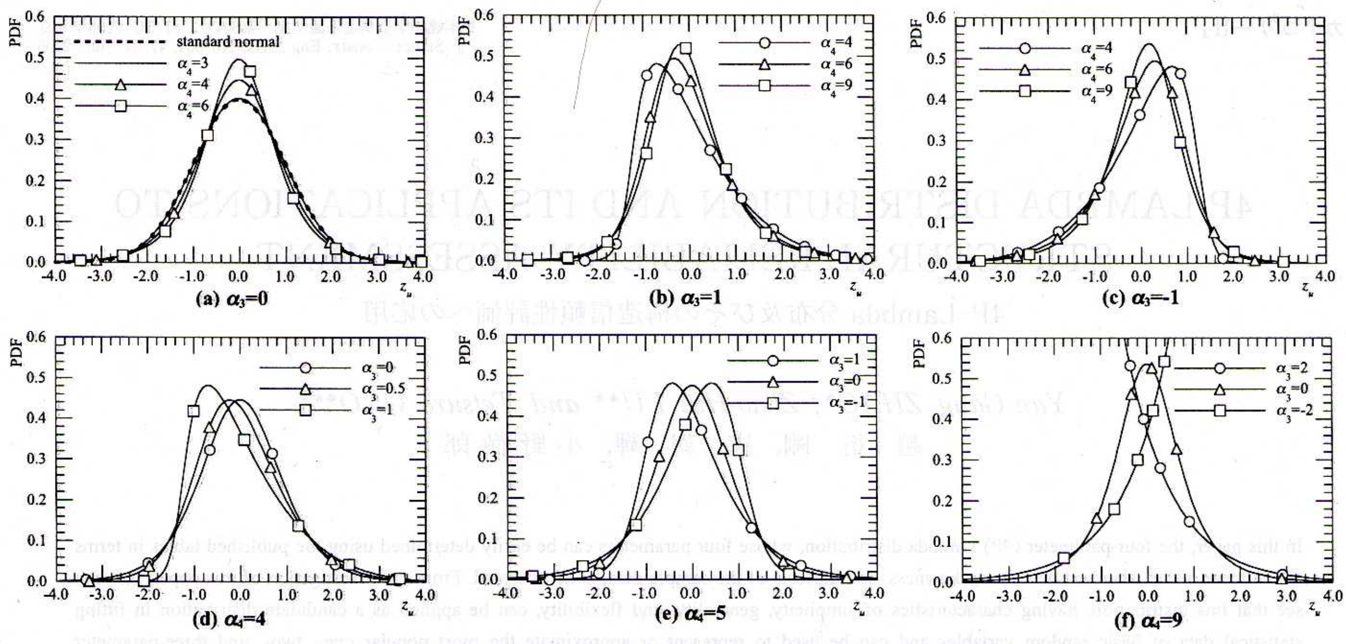

Fig. 1. The representative standard PDFs for specified α_3 and α_4 values

$$f(z) = f[R(p)] = \lambda_2\left[\lambda_3 p^{\lambda_3-1} + \lambda_4(1-p)^{\lambda_4-1}\right]^{-1}, \quad (0 \le p \le 1) \tag{2}$$

Ramberg and Schmeiser[6] showed that the kth moment of the above distribution, when it exists, is given as

$$E[z] = \lambda_1 + [1/(1+\lambda_3) - 1/(1+\lambda_4)]/\lambda_2 \tag{3a}$$

$$E[z^k] = \lambda_2^{-k}\sum_{i=0}^{k}\binom{k}{i}(-1)^i\beta(\lambda_3(k-i)+1, \lambda_4 i+1), \; k>1 \tag{3b}$$

where β denotes the beta function,

$$\beta(q,r) = \int_0^1 x^{q-1}(1-x)^{r-1}dx \tag{4}$$

The kth moment does not exist when any of the arguments of the beta function are negative. Thus, the kth moment exists if and only if $min(\lambda_3, \lambda_4) > -1/k$.

Then, the first four moments, i.e., the mean μ, the variance σ^2, the third moment $\mu_3 = E(z-\mu)^3$, and the fourth moment $\mu_4 = E(z-\mu)^4$ of z are obtained as

$$\mu = E[z] = \lambda_1 + A/\lambda_2 \tag{5a}$$

$$\sigma^2 = E\left[(z-\mu)^2\right] = (B - A^2)/\lambda_2^2 \tag{5b}$$

$$\mu_3 = E(z-\mu)^3 = \left(C - 3AB + 2A^3\right)/\lambda_2^3 \tag{5c}$$

$$\mu_4 = E(z-\mu)^4 = \left(D - 4AC + 6A^2B - 3A^4\right)/\lambda_2^4 \tag{5d}$$

where

$$A = 1/(1+\lambda_3) - 1/(1+\lambda_4) \tag{5e}$$

$$B = 1/(1+2\lambda_3) + 1/(1+2\lambda_4) - 2\beta(1+\lambda_3, 1+\lambda_4) \tag{5f}$$

$$C = 1/(1+3\lambda_3) - 3\beta(1+2\lambda_3, 1+\lambda_4) + 3\beta(1+\lambda_3, 1+2\lambda_4) - 1/(1+3\lambda_4) \tag{5g}$$

$$D = 1/(1+4\lambda_3) - 4\beta(1+3\lambda_3, 1+\lambda_4) + 6\beta(1+2\lambda_3, 1+2\lambda_4) - 4\beta(1+\lambda_3, 1+3\lambda_4) + 1/(1+4\lambda_4) \tag{5h}$$

The third dimensionless moment α_3, i.e., the skewness and the fourth dimensionless moment α_4, i.e., the kurtosis are given by

$$\alpha_3 = \mu_3/\sigma^3 \tag{5i}$$

$$\alpha_4 = \mu_4/\sigma^4 \tag{5j}$$

Apparently, the skewness and kurtosis are functions of λ_3 and λ_4, but do not depend upon λ_1 and λ_2.

2.2 Parameter Estimation

Several methods for estimating the parameters of the 4P-Lambda distribution have been proposed. These include the moment matching method[6, 7], least squares method[8], starship method[9], and randomized restart method[10]. However, all the methods for directly estimating the parameters are complicated. Fortunately, the published tables[7], which are based on Eq. 5 facilitating parameter estimation using the first four sample moments, are available and convenient. In the present paper, we will use the published tables to estimate the parameters with the first four sample moments. For convenience, a part of the published tables are provided in Appendix A.

The values of λ_1, λ_2, λ_3, and λ_4 are given in the published tables for selected values of α_3 and α_4 with $\mu=0$ and $\sigma=1$. If the values of μ, σ, α_3, and α_4 are known, the lambda values are determined from the published tables using the α_3 and α_4 values as entry points. One simply picks the values of λ_3 and λ_4 for which the α_3 and α_4 are closest to the desired values. If α_3 is negative, one uses its absolute value, and after finding the values of λ_3 and λ_4, interchanges their values and changes the sign of λ_1. (The density with a skewness of $-\alpha_3$ is the mirror image of that with a skewness of α_3.)

Since the λ_1 and λ_2 values given in the published tables are for a variate with $\mu=0$ and $\sigma=1$, multiplying the resulting variate by σ and adding μ to it achieves the desired result. This reduces to computing

$$\lambda_1(\mu,\sigma) = \lambda_1(0,1)\sigma + \mu \tag{6a}$$

$$\lambda_2(\mu,\sigma) = \lambda_2(0,1)/\sigma \tag{6b}$$

2.3 Representative PDFs of the Distribution

Once the four parameters are determined, the probability density curves can be plotted with aid of Eqs. 1 & 2 for values of p ranging from zero to one. That is, $f[R(p)]$ is plotted on the y-axis versus $R(p)$ on the x-axis.

The representative PDFs of this distribution includes a wide range of curve shapes as illustrated by the standard density plots in Fig. 1. One can see that the PDFs reflect the skewness and kurtosis quite well. And one can also see that the left tail of PDF is long for negative α_3 and the right tail is long for positive α_3. This characteristic is especially important when the fourth-moment reliability index based on this distribution is used in structural reliability assessment as described later. Especially, when α_3=0

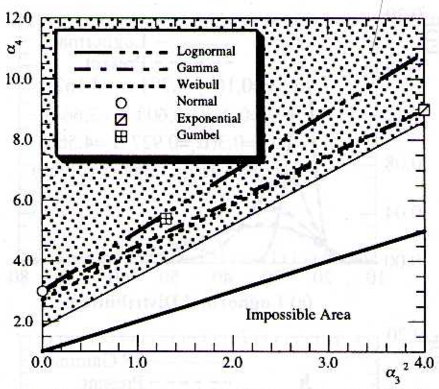

Fig. 2. Operable area of 4P-Lambda distribution

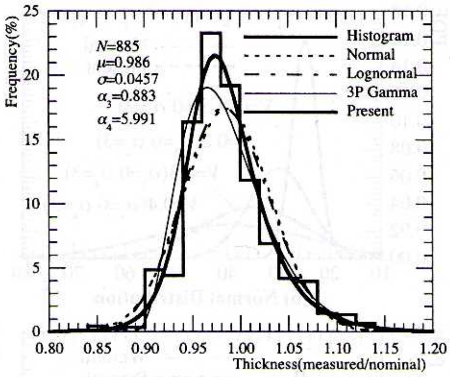

Fig. 3. Data Fitting for Thickness

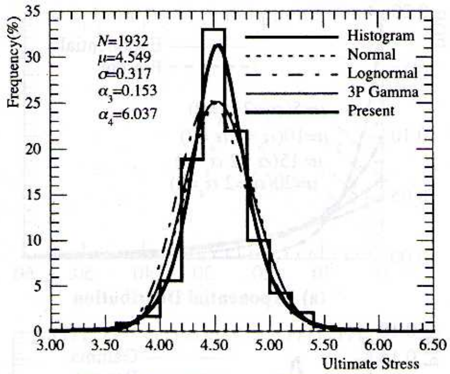

Fig. 4. Data Fitting for Ultimate Stress

and α_4=3, the distribution results in an approximation to the standard normal distribution for which $max|\Phi(z_u)-R^{-1}(z_u)|\approx.001$, where $\Phi(z_u)$ is the standard normal distribution function. The PDFs of the two distributions are also shown in Fig. 1a. One can clearly see that the 4P-Lambda distribution approximately approaches the normal distribution. Schmeiser[11] has shown that the limiting distribution of this distribution is exponential with parameter θ as $\lambda_4 \to 0$ when $\lambda_1 = \lambda_3 = 0$ and $\lambda_2 = \lambda_4/\theta$.

2.4 Operable Area of the Distribution in the α_3^2-α_4 Plane

As described previously, the values of parameters obtained using the published tables are based on Eq. 5. For a specified value of α_3, when the values of α_4 are below a limit value, the Eq. 5 will become inoperable. Using the limit value of α_4 for which Eq. 5 is inoperable corresponding to the selected α_3, a lower boundary line in the α_3^2-α_4 plane can be depicted as shown in Fig. 2, in which the operable area of the distribution is indicated by the shade region. The lower boundary line for which Eq. 5 is operable is found to be nearly a straight line approximately expressed by

$$\alpha_4 = 1.8 + 1.7\alpha_3^2 \tag{7}$$

In Fig. 2, the limit for all distributions[12] expressed as $\alpha_4=\alpha_3^2+1$ is also depicted along with α_3^2-α_4 relationship for some commonly used distributions, i.e., the normal, Gumbel, and the exponential distribution, which are represented by a single point, the lognormal, the Gamma, and the Weibull distributions, which are represented by a line. One can see that the operable area of the 4P-Lambda distribution covers a large area in the α_3^2-α_4 plane, and the α_3^2-α_4 relationship for most commonly used distributions are in the operable area of this distribution. This implies that the 4P-Lambda distribution is generally operable for common engineering use.

3. APPLICATION TO DATA ANALYSIS

3.1 Statistical Data Fitting

In order to investigate the efficiency of the 4P-Lambda distribution in fitting statistical data of a random variable, the following two examples use the practical data of H-shape structural steel collected by Ono et.al[13]. The fitting results of the histogram of the ratio between measured values and nominal values of the thickness are shown in Fig. 3, in which the number of data is 885 and the first-four moments of the data are obtained as μ=0.986, σ=0.0457, α_3=0.883, and α_4=5.991. In Fig.3, the PDFs of the normal and lognormal distributions, with the same mean value and standard deviation as the data, the PDF of the 3P-Gamma distribution whose mean value, standard deviation, and skewness are equal to those of the data, and the PDF of the 4P-Lambda distribution whose mean value, standard deviation, skewness, and kurtosis are equal to those of the data, are depicted. Fig. 3 reveals the following:

(1) The PDFs of the normal distribution and lognormal distribution have the greatest differences from the histogram of the statistical data among the four distributions. Since the normal distribution is a symmetrical distribution with the skewness=0.0 and the kurtosis=3.0, respectively, it obviously cannot be used to fit the histogram that has such a large skewness (0.883) and kurtosis (5.991), respectively. Although the lognormal distribution can reflect skewness and kurtosis in some degree, the skewness and kurtosis of the lognormal distribution are dependent on the coefficient of variation. Since the coefficient of variation for this example is very small (0.0463), the skewness and kurtosis of lognormal distribution corresponding to this coefficient of variation are too small (α_3= 0.139, α_4=3.034) to match those of the data.

(2) Since the first three moments of the 3P Gamma distribution are equal to those of the data, it fits the histogram much better than the normal and lognormal distributions. However, the kurtosis of this distribution is depending on the skewness. The kurtosis corresponding the skewness of the data is obtained as 4.17, which is too small to match that of the data.

(3) The first four moments of the 4P-Lambda distribution can be equal to those of the data, and thus can fit the histogram much better than the normal, lognormal, and 3P-Gamma distributions.

Results of the Chi-square tests of the four distributions are listed in Table 1, in which the goodness-of-fit tests were obtained using the following equation[4]

$$T = \sum_{i=1}^{k}(O_i - E_i)^2/E_i \tag{8}$$

where O_i and E_i are the observed and theoretical frequencies, respectively, k is the number of intervals used, and T is a measure of the respective goodness-of- fit. From Table 1, one can see that the results of goodness-of-fit test of the introduced distribution is T=29.1 which is much smaller than those of other distributions.

Similarly, the fitting results of the histogram of the ultimate stress are shown in Fig. 4, in which the number of data is 1932 and the first-four moments of the data are obtained as μ=4.549, σ=0.317, α_3=0.153, and α_4=6.037. From Fig. 4, one can see that since the skewness of the data is quite small, the 3P-Gamma distribution cannot show significant improvement upon the normal and lognormal distributions, whereas the 4P- Lambda can effectively fit the histograms of the available data. Also, from Table 2, the results of goodness-of-fit test verify that the introduced distribution has the best fit with T=19.34 among all the distributions.

From the above examples, one can clearly see that since the first four moments of the 4P-Lambda distribution are equal to those of the statistical data, it fits the histogram much better than the normal, the lognormal, and the 3P-Gamma distribution. This is to say, the 4P-Lambda distribution is more suitable for fitting statistical data.

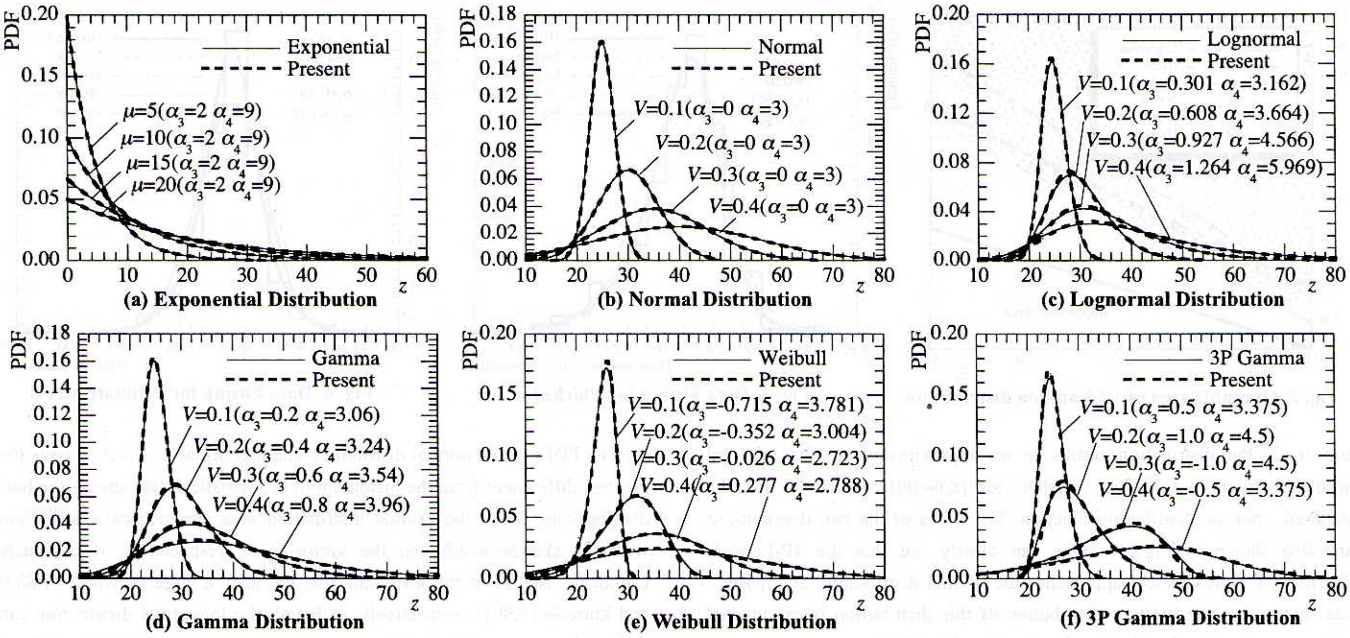

Fig. 5. PDF comparisons with some one-, two- and three-parameter distributions

Table 1. Results of test for thickness

Intervals	Freqency	Goodness of fit			
		Normal	Lognormal	3P-Gamma	Present
<0.92	54	2.12	1.11	16.86	4.35
0.92-0.94	39	15.98	18.23	34.76	12.30
0.94-0.96	145	9.06	6.23	0.42	0.45
0.96-0.98	206	26.38	23.01	9.42	2.18
0.98-1.00	170	1.96	2.16	4.20	0.00
1.00-1.02	105	6.20	4.70	0.19	1.42
1.02-10.4	61	13.48	10.95	2.45	1.60
1.04-1.06	37	7.84	6.67	2.09	0.41
1.06-1.08	35	1.24	1.11	1.86	6.15
>1.08	13	13.48	7.46	0.00	0.23
Sum	885	97.74	81.64	72.26	29.1

Table 2. Results of test for ultimate stress

Intervals	Freqency	Goodness of fit			
		Normal	Lognormal	3P-Gamma	Present
<4.0	64	3.38	0.18	0.71	1.09
4.0-4.2	108	29.64	35.19	33.84	5.22
4.2-4.4	365	0.29	0.43	0.12	0.76
4.4-4.6	638	57.72	51.70	53.63	6.56
4.6-4.8	424	0.05	0.42	0.18	2.79
4.8-5.0	193	19.28	12.04	13.90	1.47
5.0-5.2	82	7.53	6.86	7.17	0.10
>5.2	58	9.75	1.78	3.07	1.35
Sum	1932	127.64	108.6	112.62	19.34

3.2 Approximation for One-, Two-, and Three-Parameter Distributions

The 4P-Lambda distribution, as defined in Eq. 1, can be used to represent or approximate one-, two-, and three-parameter distributions by equating the respective first four moments. This is illustrated with the one-parameter exponential distribution, two-parameter distributions including normal, lognormal, Gamma, Weibull distributions, and three-parameter Gamma distribution. Fig. 5 shows the PDFs of the above distributions depicted as thin solid lines; in these same figures the respective 4P-Lambda distribution with the same first four moments as those of the corresponding one-, two- and three-parameter distributions, are depicted as thick dash lines. In these figures, all the two- and three-parameter distributions are shown with the mean values of μ=25, 30, 35, and 40, and coefficients of variation V=0.1, 0.2, 0.3, and 0.4.

Fig. 5 shows that the thick dash lines coincide closely with the thin solid lines, demonstrating the flexibility of the 4P-Lambda distribution for representing one-, two-, and three-parameter distributions. This flexibility can be useful in the structural reliability analysis as described below.

4. APPLICATION TO STRUCTURAL RELIABILITY ASSESSMENT

4.1 The Fourth-Moment Reliability Index

For a performance function $z=G(\mathbf{X})$, if the first four moments of $G(\mathbf{X})$ are obtained, the probability of failure, $P(G\leq 0)$, can be estimated by assuming $G(\mathbf{X})$ obeys the 4P-Lambda distribution.

For the standardized random variable z_u

$$z_u = (z - \mu_G)/\sigma_G \tag{9}$$

since

$$P_f = P[z \leq 0] = P[z_u \leq -\mu_G/\sigma_G] = P[z_u \leq -\beta_{2M}] \tag{10}$$

where μ_G and σ_G are the mean value and standard deviation of $z=G(\mathbf{X})$, respectively, β_{2M} is the 2nd-moment (2M) reliability index, and P_f is the probability of failure.

According to Eq. 1 and Eq. 9, the standard form of the 4P-Lambda distribution defined by its inverse cumulative distribution function can be expressed as

$$z_u = R_S(p) = \lambda_1(0,1) + \left[p^{\lambda_3} - (1-p)^{\lambda_4}\right]/\lambda_2(0,1) \tag{11}$$

The fourth-moment (4M) reliability index based on the 4P-Lambda distribution can be given as

$$\beta_{4M} = -\Phi^{-1}\left[R_S^{-1}(-\beta_{2M})\right] \tag{12a}$$

$$P_f = \Phi(-\beta_{4M}) \tag{12b}$$

where β_{4M} is the fourth-moment (4M) reliability index, and Φ is the CDF of a standard normal random variable. Although Eq. 12 is not in explicit form, it can be computed easily using the published tables.

As described earlier, the left tail of PDF is long for negative α_{3G} and the right tail is long for positive α_{3G}. Since the failure probability is integrated in left tail according to Eq. 10, it is easy to understand that the fourth moment method is more suitable for negative α_{3G} than positive α_{3G}. It may be an interesting thing that for most structural reliability assessment problem, the skewness of the performance function is negative as shown in the later examples.

4.2 Computation of the Moments of the Performance Function G(X)

In order to conduct structural reliability analysis using the fourth-moment reliability index, one should firstly compute the moments of the performance function.

A common encountered performance function in structural reliability is a linear sum of independent random variables in the original space:

$$G(X) = \sum_{i=1}^{n} a_i x_i \tag{13}$$

where x_i, i=1, ..., n are mutually independent random variables and a_i, i=1, ..., n are coefficients.

The first four moments of Eq. 13 are as follows

$$\mu_G = \sum_{i=1}^{n} a_i \mu_i \tag{14a}$$

$$\sigma_G^2 = \sum_{i=1}^{n} a_i^2 \sigma_i^2 \tag{14b}$$

$$\alpha_{3G}\sigma_G^3 = \sum_{i=1}^{n} \alpha_{3i} a_i^3 \sigma_i^3 \tag{14c}$$

$$\alpha_{4G}\sigma_G^4 = \sum_{i=1}^{n} \alpha_{4i} a_i^4 \sigma_i^4 + 6\sum_{i=1}^{n-1}\sum_{j>i}^{n} a_i^2 a_j^2 \sigma_i^2 \sigma_j^2 \tag{14d}$$

where μ_i (μ_G),σ_i (σ_G), α_{3i} (α_{3G}), and α_{4i} (α_{4G}) are the mean value, standard deviation, skewness, and kurtosis of x_i (G(X)), respectively.

Another common encountered function in structural reliability is the product of independent random variables in the original space:

$$G(X) = \prod_{i=1}^{n} x_i \tag{15}$$

The first four moments of Eq. 15 are given as

$$\mu_G = \prod_{i=1}^{n} \mu_i \tag{16a}$$

$$\sigma_G^2 = \mu_G^2\left[\prod_{i=1}^{n}\left(1+V_i^2\right)-1\right] \tag{16b}$$

$$\alpha_{3G} = \left[\prod_{i=1}^{n}\left(\alpha_{3i}V_i^3 + 3V_i^2 + 1\right) - 3\prod_{i=1}^{n}\left(1+V_i^2\right)+2\right]\Big/V_G^3 \tag{16c}$$

$$\alpha_{4G} = \left[\prod_{i=1}^{n}(\alpha_{4i}V_i^4 + 4\alpha_{3i}V_i^3 + 6V_i^2 + 1) - 4\prod_{i=1}^{n}(\alpha_{3i}V_i^3 + 3V_i^2 + 1) + 6\prod_{i=1}^{n}(1+V_i^2) - 3\right]\Big/V_G^4 \tag{16d}$$

where V_i, V_G are the coefficients of variation of x_i and G(X) respectively.

For complicated and implicit performance functions including those corresponding to correlative random variables, point estimates[14] method will be used for moment computation.

4.3 Numerical Examples

In order to investigate the efficiency of the suggested fourth-moment reliability index, several examples are examined under different conditions.

Example 1.

Consider the following performance function of a simple structural column

$$G(X) = Ax_1x_2 - x_3 \tag{17}$$

where A is the nominal section area, x_1 is a random variable representing the uncertainty of A, x_2 is the yield stress, and x_3 is the compressive stress. Assume the column has an H-shape and is made of structural steel with a H300×200[15] section and having an area A=72.38cm^2, and a material of SS41[16]. The CDFs of x_1 and x_2 are unknown, the only information about them are their first four moments[13], i.e., μ_1=0.990, σ_1=0.051, α_{31}=0.709, α_{41}=3.692, μ_2=3.055t/cm^2, σ_2=0.364, α_{32}= 0.512, and α_{42}= 3.957. x_3 is assumed as a lognormal variable with mean value μ_3=100t and standard deviation σ_3=40t.

The skewnees and kurtosis of x_3 can be soon obtained as α_{33}=1.264, α_{43}=5.969. Since the first four moments of x_1, x_2, and x_3 are known, using Eqs.14 &16, the first four moments of G(X) can be easily obtained as μ_G=118.910, σ_G=49.085, α_{3G}=-0.578, and α_{4G}=4.41. The 2M reliability index is readily obtained as β_{2M}=2.423. Using the suggested formula in the present paper, the 4M reliability index is readily obtained as β_{4M}=2.074. The corresponding probability of failure is equal to 0.01905.

Using Eq. 1, the random sampling of x_1 and x_2 can be easily generated without using their CDFs and Monte-Carlo Simulation (MCS) can be thus easily conducted. The probability of failure of this performance function is obtained as P_f= 0.0188 and the corresponding reliability index is equal to 2.079 when the number of samplings is taken to be 10,000. One can see the 4M method is in close agreement with the MCS result.

Example 2.

The second example is an elastoplastic frame structure with six stories and three bays as shown in Fig. 6, with the probabilistic member strength and load listed in Table 3. The most likely failure model of this structure is also shown in Fig. 6. The corresponding performance function is

$$G(X) = 2M_1 + 2M_4 + 2M_7 + 2M_{10} + 2M_{13} + M_{14} + M_{15} - 3.8S_2 - 7.6S_3 - 11.4S_4 - 15.2S_5 - 19S_6 \tag{18}$$

Because all of the random variables in the function above have a known PDF (CDF), the reliability index can be readily obtained using First-Order Reliability Method (FORM). The FORM reliability index is β_f=3.100, which corresponds to a failure probability of P_f= 0.000968.

The skewness and kurtosis of the variables of member strength and load are also listed in Table 3. With aid of Eq. 14, the mean value, standard deviation, skewness, and kurtosis of G(X) are readily obtained as μ_G=619, σ_G=154.285, α_{3G}=-0.694, and α_{4G}=4.084. The 2M reliability index is readily obtained as β_{2M}=4.012. Using the suggested formula in the present paper, the 4M reliability index is readily obtained as β_{4M}=2.976. The corresponding probability of failure is equal to 0.00146.

Using MCS with 500,000 samples, the probability of failure for this performance function is obtained as 0.001598 with corresponding reliability index of β=2.948. One can see that the probability of failure obtained using the proposed method is closer to the result of MCS than that of FORM for this example.

Table 3. Random Variables in Example 2

Variables (independent and Lognormal)	Mean	Coefficient of variation	Skewness	Kurtosis
M_1, M_4, M_7, M_{17}, M_{18}	90.8t-m	0.1	0.301	3.1615
M_2, M_3, M_5, M_6	145.2t-m	0.1	0.301	3.1615
M_8, M_9, M_{21}, M_{22}	145.2t-m	0.1	0.301	3.1615
M_{10}, M_{13}, M_{16}	103.4t-m	0.1	0.301	3.1615
M_{11}, M_{12}, M_{14}	162.8t-m	0.1	0.301	3.1615
M_{15}, M_{19}, M_{20}	162.8t-m	0.1	0.301	3.1615
S_1	2.5t	0.4	1.264	5.969
S_2	5.0t	0.4	1. 264	5.969
S_3	7.5t	0.4	1.264	5.969
S_4	10.0t	0.4	1.264	5.969
S_5	12.5t	0.4	1.264	5.969
S_6	15.0t	0.4	1.264	5.969

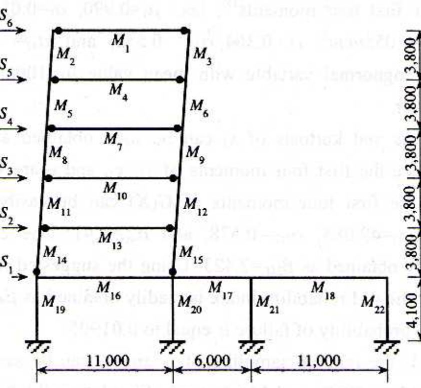

Fig. 6. Most likely failure mode of a six-story three-bay frame

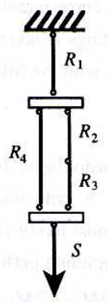

Fig. 7. Brittle Chain System

Example 3.

Considers a parallel-chain system as shown in Fig. 7. Assume that the individual components of the structural system will fail through fracture, and thus it is a brittle system (non-series).

There are two failure modes of the parallel-chain system shown in Fig. 7 with respective performance functions listed in Eq. 19.

$$g_1 = R_1 - S \qquad (19a)$$

$$g_2 = \max\{min(R_2,R_3,R_4) - S/2, \max[(min(R_2,R_3),R_4] - S\} \qquad (19b)$$

The performance function of the system can be defined as the minimum of the above; i.e.,

$$G(X) = min\{g_1, g_2\} \qquad (19c)$$

where the facture strengths R, and load S are independent lognormal random variables with means and standard deviations of μ_{R1}=2200kg, μ_{R2}=2100 kg, μ_{R3}=2300 kg, μ_{R4}=2000 kg, μ_S=1200 kg, σ_{R1}=220 kg, σ_{R2}=210 kg, σ_{R3}=230 kg, σ_{R4}=20 kg, and σ_S=240 kg.

Using the point estimates method[14], the first four moments of $G(X)$ are approximately as μ_G=993.360, σ_G =316.836, α_{3G}=-0.275, and α_{4G}=3.109. The second moment reliability index are easily obtained as β_{2M}=3.135. Using the presented formula in the present paper, the 4M reliability index is readily obtained as β_{4M}=2.872. The probability of failure corresponding to the 4M reliability index is equal to 2.04×10^{-3}.

Using MCS with 500,000 samples, the probability of failure for this system is obtained as 2.506×10^{-3} with corresponding reliability index of β=2.806. One can see that the result of the proposed method is in close agreement with the MCS result, whereas the second-moment approximation is significantly in error.

5. CONCLUSIONS

The 4P-Lambda distribution is investigated, and its applications are emphasized including statistical data analysis and structural reliability assessment. It is found that

(1) The 4P-Lambda distribution has a single expression, and the four parameters can be easily determined with the first four central moments using the published tables.

(2) The 4P-Lambda distribution is generally operable for common engineering use.

(3) With four parameters, the 4P-Lambda distribution has more flexibility for fitting statistical data of basic random variables, and can more effectively fit the histograms of available data than two-parameter or three-parameter distributions.

(4) The 4P-Lambda distribution can be used to represent or approximate some popular distributions, such as one-parameter exponential distribution, two-parameter distributions including normal, lognormal, Gamma, Weibull distributions, and three-parameter Gamma distribution and so on.

(5) For some performance functions, if the first four moments are obtained, the 4P-Lambda distributions can be conveniently applied to obtain a moment-based reliability index.

(6) The structural reliability evaluation can be conducted with aid of the suggested 4M reliability index even when the CDFs or PDFs of the basic random variables are unknown.

It should be noted that compared to the determination of mean value and standard deviation, more sample data is required to determine the values of high order moments. Since the first four moments have clear physical meanings, the 4P-Lambda distribution is investigated in this paper. Further study is required to understand the relationship between the accuracy of high order moment and sample size.

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Appendix A Lambda parameters for given values of *skewness* (α_3) *and kurtosis* (α_4) when μ=0 and σ=1.

The table listed here is a part of the table constructed by Ramberg et.al[7]. The parameter values given in this table are for a variate with μ=0 and σ=1. The procedure for adjusting the parameters to reflect a different mean or variance is given in Section 2.2. A plus sign (+) next to a tabled value indicates that the value has two leading zeroes and should be multiplied by 10^{-2}. Similarly, a dollar sign($) next to a tabled value indicates that the should be multiplied by 10^{-4}.

α_3=0.00

α_4	λ_1	λ_2	λ_3	λ_4
2.8	.0	.2433	.1765	.1765
3.0	.0	.1974	.1349	.1349
3.2	.0	.1563	.1016	.1016
3.4	.0	.1191	.0742	.0742
3.6	.0	.0852	.0512	.0512
3.8	.0	.0545	.0317	.0317
4.0	.0	.0262	.0148	.0148
4.1	.0	.0128	.7140+	.7140+
4.2	.0	-.0659+	-.0363+	-.0363+
4.3	.0	-.0123	-.6706+	-.6706+
4.4	.0	-.0241	-.0130	-.0130
4.6	.0	-.0466	-.0246	-.0246
4.8	.0	-.0676	-.0350	-.0350
5.0	.0	-.0870	-.0443	-.0443
5.2	.0	-.1053	-.0528	-.0528
5.4	.0	-.1277	-.0606	-.0606
5.6	.0	-.1389	-.0677	-.0677
5.8	.0	-.1541	-.0742	-.0742
6.0	.0	-.1686	-.0802	-.0802

α_3=0.10

α_4	λ_1	λ_2	λ_3	λ_4
3.0	-.117	.1977	.1205	.1503
3.2	-.092	.1572	.0936	.1111
3.4	-.076	.1203	.0698	.0803
3.6	-.065	.0866	.0490	.0552
3.8	-.057	.0558	.0308	.0342
4.0	-.049	.0276	.0149	.0163
4.1	-.048	.0142	.7606+	.8302+
4.2	-.046	.1440+	.0762+	.0828+
4.3	-.044	-.0109	-.5703+	-.6174+
4.4	-.041	-.0227	-.0118	-.0127
4.6	-.037	-.0452	-.0231	-.0247
4.8	-.036	-.0661	-.0332	-.0354
5.0	-.033	-.0857	-.0424	-.0450
5.2	-.032	-.1040	-.0507	-.0537
5.4	-.030	-.1213	-.0584	-.0616
5.6	-.028	-.1375	-.0654	-.0688
5.8	-.027	-.1530	-.0719	-.0755
6.0	-.027	-.1674	-.0778	-.0816
6.2	-.025	-.1811	-.0834	-.0872

α_3=0.20

α_4	λ_1	λ_2	λ_3	λ_4
3.0	-.237	.1983	.1065	.1672
3.2	-.187	.1599	.0866	.1230
3.4	-.154	.1240	.0677	.0889
3.6	-.132	.0908	.0482	.0615
3.8	-.116	.0601	.0314	.0389
4.0	-.103	.0318	.0164	.0198
4.1	-.097	.0185	.9467+	.0113
4.2	-.093	.5707+	.2894+	.3429+
4.3	-.089	-.6641+	-.3342+	-.3929+
4.4	-.085	-.0185	-.9261+	-.0108
4.6	-.079	-.0410	-.0202	-.0233
4.8	-.074	-.0622	-.0302	-.0345
5.0	-.069	-.0818	-.0392	-.0444
5.2	-.065	-.1003	-.0475	-.0534
5.4	-.061	-.1176	-.0551	-.0615
5.6	-.058	-.1339	-.0621	-.0689
5.8	-.055	-.1494	-.0686	-.0757
6.0	-.053	-.1639	-.0745	-.0819
6.2	-.051	-.1778	-.0801	-.0877

α_3=0.30

α_4	λ_1	λ_2	λ_3	λ_4
3.0	-.362	.1991	.0925	.1859
3.2	-.288	.1641	.0796	.1377
3.4	-.239	.1298	.0640	.1003
3.6	-.204	.0973	.0481	.0704
3.8	-.179	.0671	.0330	.0460
4.0	-.160	.0389	.0190	.0255
4.2	-.144	.0127	.6175+	.8035+
4.3	-.138	.0789+	.0380+	.0489+
4.4	-.131	-.0116	-.5554+	-.7057+
4.5	-.129	-.0231	-.0110	-.0139
4.6	-.121	-.0343	-.0163	-.0203
4.8	-.113	-.0554	-.0260	-.0319
5.0	-.105	-.0752	-.0350	-.0423
5.2	-.100	-.0939	-.0432	-.0517
5.4	-.094	-.1114	-.0508	-.0601
5.6	-.089	-.1279	-.0578	-.0678
5.8	-.085	-.1435	-.0643	-.0748
6.0	-.081	-.1582	-.0703	-.0812
6.2	-.078	-.1722	-.0759	-.0872

α_3=0.40

α_4	λ_1	λ_2	λ_3	λ_4
3.0	-.494	.2000	.0782	.2069
3.2	-.400	.1690	.0718	.1555
3.4	-.333	.1371	.0609	.1149
3.6	-.284	.1060	.0482	.0824
3.8	-.248	.0764	.0351	.0558
4.0	-.222	.0485	.0223	.0337
4.2	-.200	.0224	.0103	.0149
4.3	-.190	.0100	.4597+	.6521+
4.4	-.182	-.0397+	-.0182+	-.0254+
4.5	-.174	-.0136	-.6204+	-.8533+
4.6	-.166	-.0248	-.0113	-.0153
4.8	-.155	-.0462	-.0209	-.0277
5.0	-.146	-.0662	-.0297	-.0387
5.2	-.136	-.0850	-.0379	-.0485
5.4	-.129	-.1027	-.0455	-.0574
5.6	-.122	-.1194	-.0525	-.0654
5.8	-.115	-.1352	-.0591	-.0727
6.0	-.111	-.1501	-.0651	-.0794

α_3=0.50

α_4	λ_1	λ_2	λ_3	λ_4
3.4	-.440	.1454	.0566	.1332
3.6	-.376	.1163	.0476	.0979
3.8	-.329	.0877	.0369	.0689
4.0	-.290	.0604	.0259	.0477
4.2	-.262	.0345	.0149	.0243
4.3	-.248	.0221	.9582+	.0152
4.4	-.238	.0101	-.4383+	.6815+
4.5	-.228	-.1612+	-.0700+	-.1066+
4.6	-.219	-.0128	-.5570+	-.8334+
4.8	-.202	-.0344	-.0149	-.0216
5.0	-.188	-.0546	-.0236	-.0333
5.2	-.177	-.0737	-.0317	-.0438
5.4	-.167	-.0917	-.0393	-.0532
5.6	-.157	-.1087	-.0464	-.0617
5.8	-.150	-.1246	-.0529	-.0694
6.0	-.142	-.1398	-.0591	-.0764
6.2	-.137	-.1542	-.0648	-.0829
6.4	-.131	-.1679	-.0702	-.0889

α_3=0.60

α_4	λ_1	λ_2	λ_3	λ_4
3.4	-.562	.1539	.0504	.1554
3.6	-.482	.1273	.0454	.1171
3.8	-.420	.1005	.0379	.0854
4.0	-.372	.0740	.0289	.0589
4.2	-.335	.0486	.0194	.0366
4.4	-.302	.0244	.9911+	.0175
4.5	-.289	.0128	.5215+	.8965+
4.6	-.277	.1492+	.0611+	.1025+
4.7	-.266	-.9531+	-.3916+	-.6425+
4.8	-.256	-.0202	-.8326+	-.0134
5.0	-.238	-.0407	-.0168	-.0261
5.2	-.222	-.0600	-.0248	-.0373
5.4	-.209	-.0782	-.0323	-.0474
5.6	-.197	-.0956	-.0394	-.0565
5.8	-.187	-.1118	-.0460	-.0647
6.0	-.179	-.1273	-.0522	-.0722
6.2	-.171	-.1419	-.0580	-.0790
6.4	-.163	-.1559	-.0635	-.0853

α_3=0.70

α_4	λ_1	λ_2	λ_3	λ_4
3.6	-.606	.1385	.0409	.1406
3.8	-.529	.1139	.0369	.1060
4.0	-.467	.0889	.0307	.0768
4.2	-.419	.0643	.0232	.0522
4.4	-.379	.0406	.0151	.0312
4.6	-.344	.0178	.6767+	.0130
4.7	-.331	.6799+	.2607+	.4872+
4.8	-.317	-.3917+	-.1512+	-.2750+
4.9	-.305	-.0144	-.5574+	-.9893+
5.0	-.294	-.0245	-.9565+	-.0166
5.2	-.276	-.0441	-.0173	-.0289
5.4	-.257	-.0626	-.0247	-.0398
5.6	-.243	-.0802	-.0317	-.0496
5.8	-.229	-.0967	-.0383	-.0584
6.0	-.219	-.1125	-.0445	-.0665
6.2	-.209	-.1275	-.0504	-.0738
6.4	-.199	-.1417	-.0560	-.0805
6.6	-.191	-.1554	-.0613	-.0867

α_3=0.80

α_4	λ_1	λ_2	λ_3	λ_4
3.6	-.754	.1492	.0333	.1691
3.8	-.657	.1272	.0333	.1310
4.0	-.582	.1042	.0303	.0989
4.2	-.519	.0810	.0254	.0716
4.4	-.468	.0580	.0192	.0482
4.6	-.425	.0357	.0123	.0281
4.8	-.392	.0142	.5035+	.0107
4.9	-.375	.3770+	.1352+	.2770+
5.0	-.361	-.6291+	-.2278+	-.4531+
5.1	-.349	-.0164	-.5981+	-.0116
5.2	-.335	-.0261	-.9598+	-.0181
5.4	-.313	-.0449	-.0167	-.0301
5.6	-.295	-.0626	-.0235	-.0408
5.8	-.279	-.0795	-.0300	-.0504
6.0	-.264	-.0958	-.0363	-.0592
6.2	-.251	-.1110	-.0422	-.0671
6.4	-.240	-.1255	-.0478	-.0743
6.6	-.230	-.1394	-.0531	-.0810

α_3=0.90

α_4	λ_1	λ_2	λ_3	λ_4
4.0	-.717	.1193	.0269	.1258
4.2	-.639	.0979	.0251	.0953
4.4	-.575	.0762	.0214	.0693
4.6	-.522	.0547	.0164	.0468
4.8	-.478	.0337	.0106	.0273
5.0	-.439	.0132	.4328+	.0102
5.1	-.422	.3339+	.1111+	.2526+
5.2	-.407	-.6388+	-.2154+	-.4735+
5.3	-.394	-.0159	-.5428+	-.0116
5.4	-.379	-.0252	-.8694+	-.0180
5.6	-.353	-.0432	-.0152	-.0298
5.8	-.334	-.0605	-.0215	-.0405
6.0	-.317	-.0768	-.0275	-.0500
6.2	-.301	-.0924	-.0334	-.0587
6.4	-.287	-.1073	-.0390	-.0666
6.6	-.273	-.1215	-.0444	-.0738
6.8	-.262	-.1352	-.0495	-.0805
7.0	-.252	-.1481	-.0544	-.0866

α_3=1.00

α_4	λ_1	λ_2	λ_3	λ_4
4.0	-.886	.1333	.0193	.1588
4.2	-.787	.1142	.0212	.1244
4.4	-.706	.0943	.0206	.0950
4.6	-.638	.0741	.0182	.0697
4.8	-.581	.0539	.0144	.0477
5.0	-.533	.0340	.9695+	.0285
5.2	-.492	.0146	.4383+	.0117
5.3	-.474	.5192+	.1584+	.4061+
5.4	-.445	-.0317+	-.0101+	-.0242+
5.5	-.442	-.0132	-.4176+	-.9946+
5.6	-.429	-.0222	-.7097+	-.0164
5.8	-.403	-.0395	-.0129	-.0282
6.0	-.379	-.0562	-.0187	-.0388
6.2	-.358	-.0721	-.0244	-.0484
6.4	-.341	-.0873	-.0299	-.0571
6.6	-.325	-.1019	-.0352	-.0651
6.8	-.309	-.1158	-.0404	-.0723
7.0	-.297	-.1291	-.0453	-.0790

α_3=1.10

α_4	λ_1	λ_2	λ_3	λ_4
4.4	-.869	.1117	.0157	.1267
4.6	-.781	.0932	.0165	.0977
4.8	-.708	.0743	.0154	.0727
5.0	-.647	.0552	.0128	.0508
5.2	-.596	.0365	.9168+	.0318
5.4	-.552	.0181	.4839+	.0150
5.5	-.532	.9038+	.2484+	.7342+
5.6	-.517	.0997+	.0279+	.0795+
5.7	-.497	-.8629+	-.2479+	-.6726+
5.8	-.481	-.0173	-.5046+	-.0132
6.0	-.451	-.0340	-.0103	-.0251
6.2	-.427	-.0501	-.0155	-.0358
6.4	-.403	-.0656	-.0208	-.0455
6.6	-.384	-.0805	-.0259	-.0544
6.8	-.366	-.0947	-.0309	-.0624
7.0	-.350	-.1084	-.0358	-.0698
7.2	-.335	-.1214	-.0405	-.0766
7.4	-.322	-.1341	-.0451	-.0829

α_3=1.20

α_4	λ_1	λ_2	λ_3	λ_4
5.0	-.792	.0764	.0124	.0784
5.2	-.723	.0586	.0112	.0565
5.4	-.668	.0408	.8705+	.0372
5.6	-.619	.0233	.5411+	.0202
5.7	-.597	.0146	.3525+	.0124
5.8	-.577	.6088+	.1515+	.5050+
5.9	-.558	-.2319+	-.0594+	-.1884+
6.0	-.562	-.0962+	-.0245+	-.0784+
6.2	-.508	-.0268	-.7343+	-.0206
6.4	-.481	-.0424	-.0120	-.0315
6.6	-.454	-.0575	-.0168	-.0414
6.8	-.432	-.0719	-.0215	-.0504
7.0	-.412	-.0860	-.0262	-.0587
7.2	-.394	-.0993	-.0308	-.0662
7.4	-.378	-.1123	-.0353	-.0732
7.6	-.362	-.1247	-.0397	-.0796
7.8	-.349	-.1366	-.0439	-.0856
8.0	-.337	-.1480	-.0480	-.0911

α_3=1.30

α_4	λ_1	λ_2	λ_3	λ_4
5.6	-.749	.0466	.7959+	.0447
5.8	-.695	.0300	.5783+	.0273
6.0	-.604	.0286+	.6619$	.0239+
6.1	-.617	.0446+	.0100+	.0375+
6.2	-.616	-.0526+	-.0118+	-.0442+
6.3	-.589	-.0104	-.2450+	-.8504+
6.4	-.572	-.0182	-.4399+	-.0146
6.6	-.539	-.0333	-.8469+	-.0258
6.8	-.510	-.0480	-.0127	-.0360
7.0	-.485	-.0622	-.0170	-.0453
7.2	-.463	-.0758	-.0213	-.0538
7.4	-.442	-.0890	-.0256	-.0616
7.6	-.424	-.1017	-.0298	-.0688
7.8	-.407	-.1140	-.0340	-.0754
8.0	-.392	-.1258	-.0380	-.0816
8.2	-.378	-.1372	-.0420	-.0873
8.4	-.365	-.1480	-.0458	-.0926
8.6	-.353	-.1584	-.0495	-.0975

α_3=1.40

α_4	λ_1	λ_2	λ_3	λ_4
6.0	-.782	.0379	.5603+	.0365
6.2	-.729	.0222	.3785+	.0204
6.3	-.706	.0145	.2611+	.0130
6.4	-.683	.6822+	.1292+	.5987+
6.5	-.660	-.1226+	-.0244+	-.1052+
6.6	-.643	-.8266+	-.1702+	-.6968+
6.8	-.607	-.0230	-.0560+	-.0187
7.0	-.575	-.0373	-.8670+	-.0293
7.2	-.547	-.0510	-.0124	-.0389
7.4	-.521	-.0645	-.0163	-.0478
7.6	-.498	-.0775	-.0202	-.0559
7.8	-.475	-.0900	-.0242	-.0633
8.0	-.458	-.1020	-.0280	-.0702
8.2	-.440	-.1137	-.0319	-.0766
8.4	-.423	-.1250	-.0357	-.0825
8.6	-.410	-.1358	-.0393	-.0881
8.8	-.395	-.1463	-.0430	-.0932
9.0	-.383	-.1564	-.0465	-.0980

α_3=1.50

α_4	λ_1	λ_2	λ_3	λ_4
6.0	-.957	.0622	.3907+	.0677
6.2	-.885	.0471	.4441+	.0483
6.4	-.824	.0321	.3885+	.0313
6.6	-.688	.0566+	.0104+	.0494+
6.7	-.747	.9962+	.1538+	.9059+
6.8	-.714	-.0290+	-.4897$	-.0256+
6.9	-.704	-.4446+	-.0768+	-.3882+
7.0	-.684	-.0115	-.2088+	-.9875+
7.2	-.647	-.0254	-.4989+	-.0210
7.4	-.615	-.0390	-.8156+	-.0312
7.6	-.585	-.0520	-.0115	-.0404
7.8	-.558	-.0648	-.0150	-.0489
8.0	-.536	-.0767	-.0184	-.0565
8.2	-.514	-.0891	-.0221	-.0640
8.4	-.494	-.1007	-.0257	-.0707
8.6	-.476	-.1118	-.0292	-.0769
8.8	-.459	-.1225	-.0327	-.0826
9.0	-.443	-.1330	-.0362	-.0880

和文要約

1. 序

構造信頼性解析では、外力や抵抗に含まれている不確定性は一般に確率変数として表され、確率変数の分布形を仮定・決定することは肝要なステップとなる。確率変数の分布形を決定するために、観測データにフィットする予想分布として幅広く用いられる正規、対数正規分布などの分布形はほとんど平均値と標準偏差の二つのパラメータで決められる。一旦平均値と標準偏差が決定すると、分布形の高次モーメントも決められ、分布形の重要な特徴としての歪度と尖度などは自由に選択できないことは問題点である。それを解決するために、平均値、標準偏差、歪度の三つのパラメータで決められる分布形を提示されたが、統計データの尖度を適切に反映できないことに不満が残る。本研究では統計データを精度よくフィットするために平均値、標準偏差、歪度及び尖度の四つのパラメータで決められる分布形として、4P-Lambda分布及びその構造信頼性評価への応用を考察することを試みる。

2. 4P-Lambda分布

2.1 分布の定義

4P-Lambda分布の確率密度関数(PDF)は式(2)で定義する。この分布形を有する確率変数xの4次までのモーメントは式(5)のように求められる。式(5)より、xの平均値と標準偏差はパラメータλ_1とλ_2の関数であり、歪度と尖度には平均値μと標準偏差σを含めず，歪度と尖度はパラメータλ_3とλ_4だけの関数である。即ち，4P-Lambda分布の分布形は平均値，標準偏差，歪度および尖度の四つのモーメントによって得られたパラメータλ_1, λ_2, λ_3, λ_4で決められる。

2.2 パラメータの決定

モーメントからパラメータの求め方として多く提案されたが、ここでは既存のパラメータ表(付録A参照)を利用する。

2.3 確率密度関数の図示

各種のα_3, α_4の組合に対して標準型の確率密度関数はFig.1に示す。Fig.1により、確率密度関数に歪度および尖度の影響を適切に反映していることが分かる。また、全ての確率密度関数に対して、正のα_3より負のα_3の方は左尻尾が長いことが分かる。この分布形はα_3=0, α_4=3に近付くことにつれて正規分布の分布形に近付く。

2.4 適用範囲

Fig.2に4P-Lambda分布が成立するためのα_3, α_4の範囲を示す。Fig.2により、4P-Lambda分布の適用範囲は式(7)の直線の上の影の部分であり、工学的に良く用いられる正規分布、対数正規分布、Gamma分布、指数分布等をカバーしていることが分かる。

3. データ分析への適用

3.1 実測データによる分布形の考察

H型鋼の断面に関する885個のデータ及び極限応力に関する1932個のデータをフィットする正規、対数正規分布、3P-Gamma分布および4P-Lambda分布の確率密度関数をFig. 3とFig. 4に示す。4P-Lambda分布は正規、対数正規分布及び3P-Gamma分布より明らかにこれらのデータをよくフィットすることが分かる。正規、対数正規分布、3P-Gamma分布および4P-Lambda分布に対する検定の結果をTable 1とTable 2に示す。正規、対数正規分布、3P-Gamma分布より本提案分布の適応度がかなり小さい、本提案分布は正規、対数正規分布、3P-Gamma分布より明らかにこれらのデータをよく適応していることが分かる。

3.2 既存の三つ以下のパラメータを有する分布形との比較

既存の三つ以下のパラメータを有する分布形との比較により4P-Lambda分布の一般性を検討する。平均値が5, 10, 15, 20の４ケースの指数分布、平均値が25, 30, 35, 40、変動係数が0.1, 0.2, 0.3 と0.4の４ケースの正規分布、対数正規分布、Gamma分布，Weibull分布および3P-Gamma分布の確率密度関数と4P-Lambda分布の確率密度関数の比較をそれぞれFig.5の(a), (b), (c), (d), (e), (f)に示す。これらの図により、ほとんどの場合では細い実線と太い破線はほぼ重なっていることが分かる。即ち，4P-Lambda分布は既存の三つ以下のパラメータを有する分布を含む一般的な分布として用いることができる。

4．構造信頼性解析へ応用

4.1 ４次モーメント信頼性指標の導出

式(9)のように限界状態関数 $z=G(\mathbf{X})$ を標準化し、式(10)のような破壊確率の定義により、４次モーメント信頼性指標β_{4M}は式(12)のように得られる。破壊確率は限界状態関数のPDFの左尻尾から$-\beta_{2M}$までの積分から得られるので、正のα_{3G}より負のα_{3G}（左尻尾が長い）の限界状態関数に対してより適切な結果を得ることができる。

4.2 限界状態関数のモーメントの算出

式(13)のような独立確率変数の線形和の限界状態関数に対して、$z=G(\mathbf{X})$の４次までのモーメントは式(14)のように算出され、式(15)のような独立確率変数の乗積の限界状態関数に対して、$z=G(\mathbf{X})$の４次までのモーメントは式(16)のように算出される。さらに複雑の限界状態関数のモーメントは点推定法によって得られる。

4.3 例題

例題1, 式(17)の限界状態関数に対して、x_1, x_2の分布形が分からなく，４次までのモーメントのみが分かることにする。x_3は対数正規確率変数である。式(14)と(16)の組み合わせにより、限界状態関数の４次までのモーメントは μ_G=118.910, σ_G=49.085, α_{3G}=-0.578, α_{4G}=4.41のように得られる。２次モーメント信頼性指標はβ_{2M}=2.423ように得られ、式(12)により４次モーメント信頼性指標はβ_{4M}=2.074になる。さらに、式(1)によりxの乱数が容易に発生でき、MCSにも適用できる。サンプル数が10,000で得られたMCSの結果はP_f=0.0188となり、対応する信頼性指標は2.079であり、４次モーメント信頼性指標とほぼ一致することが分かる。

例題2, Fig.6に示す６層３スパン骨組の崩壊モードの限界状態関数は式(18)に示す。MCSの結果と４次モーメント信頼性指標とほぼ一致することが分かる。例題3にも同じような結果が得られた。

5. まとめ

1. 4P-Lambda分布は予想分布として応用することができ、既存の三つ以下のパラメータを有する分布形より統計データをよく対応することが判る。
2. 4P-Lambda分布は既存のGamma、Weibull、対数正規分布などの既存の三つ以下のパラメータを有する分布形を代表することができる。
3. 4P-Lambda分布より、４次モーメント信頼性指標が容易に得られ、構造信頼性解析に応用することができる。
4. 4P-Lambda分布の分布形が単一の表現式でありながらも、工学的に良く用いられる正規分布、対数正規分布、Gamma分布、指数分布等のα_3^2-α_4範囲をカバーしている。

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