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LIKELY STORY MECHANISMS OF STEEL FRAME STRUCTURES

鉄骨骨組の層崩壊機構に関する確率論的考察

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The subject of failure mechanism is of much importance not only for structural reliability analysis but also for structural design. In the present paper the preferable and undesirable failure modes in aseismic design of steel frame structures are analyzed, and the failure probabilities of the story mechanisms of frame structure are investigated through both computational and theoretical analysis. The story mechanisms are classified into three types: upper collapse pattern, middle collapse pattern and lower collapse pattern. It is found that: the upper story mechanism with a relatively smaller number of failure stories is less likely to develop; for middle story mechanisms with an identical number of failure stories, the reliability index increases with the increase of the number of intact stories below the failure stories of structure; and the occurrence probability of lower story mechanism is always higher than that of middle story mechanisms with the same number of failure stories.

Keywords: *frame structure, story mechanism, failure probability, reliability analysis.*
ラーメン骨組、層崩壊機構、破壊確率、信頼性解析

1. INTRODUCTION

Failure mechanisms of frame structures are generally concerned not only in structural reliability analysis but also in structural design. In deterministic design of frame structures, some preferred failure modes are often selected and the strength of structural members are designed according to the strength requirements of selected failure modes. In reality, however, the designed structure may collapse unexpectedly according to some undesirable failure modes due to the uncertainties in member strength and external load[1), 2)]. Since it is impossible to absolutely ensure the structure collapses according to the designed failure mode in deterministic meaning, it is essential to identify the likely failure modes and understand the order in which they are likely to occur.

Many studies on the collapse modes of frames have been performed so far, and basically either the deterministic way or the probabilistic way was employed in the analysis. In the studies of deterministic way, Nakashima et al. investigated earthquake responses of steel moment frame and the effect of column over-design factor (COF) on the formation of specific failure modes[3), 4)]. In a research on the optimum elastic limit strength distribution of structural members by Ogawa[5),6)], it was indicated that the plastic hinges tend to form in more stories from the first story with the increase of the COF, and to ensure an entire beam-hinging pattern, a quite high COF is required. Akiyama[7)] and Ogawa et al.[8)] each proposed a distribution law to predict the damage distribution along the height of frame structure, and the analysis on the effect of the strength and stiffness of structural members on the formation of each collapse types was conducted in Akiyama's research. As for the studies in probabilistic way, the method of identifying stochastically dominant failure modes by Zimmerman[9)] is to identify the failure modes based on their contribution to the system probability of failure after getting the solution of a series of stochastic mathematical programs; Ang and Ma[10)] developed a method to find the stochastically relevant mode directly by solving a nonlinear optimization problem, which was performed to find the minimal reliability index; Ohi[11)] developed the stochastic limit analysis method, which is one of the mathematical programming techniques to obtain the likely failure modes in relatively short computation time.

In the present paper, the preferable and undesirable failure modes of frame structures are investigated first, and the followed examination focuses particularly on story mechanism, one kind of failure pattern of frame that should be avoided during aseismic excitation. The probabilistic orders of story mechanisms of frame structures designed under a certain reliability level are investigated through computing and comparing their reliability indices. The likely story mechanisms of steel frame structures are identified.

2. BASIC ASSUMPTIONS

For the ductile frame structures considered in this paper, all the computations are conducted on the basis of the following commonly used assumptions:

- Frame structures have an elastic-plastic behavior. The failure of a section means the imposition of a hinge and an artificial moment at this section.
- Within the structural parameters, only the member strength is considered as a random variable, the others are assumed to be deterministic for a specific structure.
- Geometrical second-order and shear effects are neglected. The effect of axial forces on the reduction of moment capacities is also neglected.
- The stationary earthquake load inverse triangularly distributed along the height of the structure is adopted in the computation.

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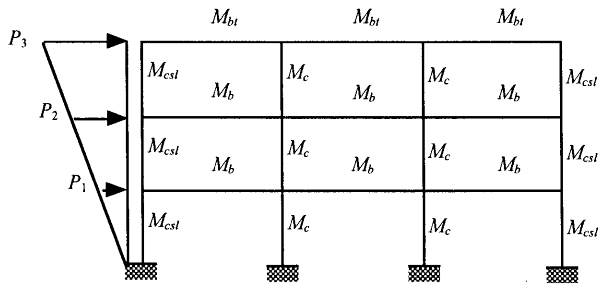


Fig. 1. 3-story-3-bay Frame Subjected to Earthquake

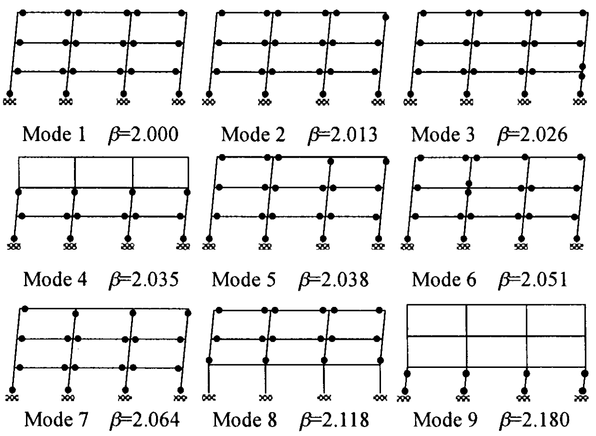


Fig. 2. The Most Likely Failure Modes for a 3-story-3-span Frame

3. INVESTIGATION OF FAILURE LEVELS

Consider a 3-story-3-bay frame structure subjected to triangularly distributed earthquake load, as shown in Fig. 1, in which all the column-beam nodes are designed with the same COF (denoted by variable "C_{of}") value, the member strengths can be designed according to Eq. (1).

$$\mu_{bij} = 2\mu_b,\ \ \mu_{csl} = C_{of}\mu_b,\ \ \mu_{cl} = 2C_{of}\mu_b \tag{1}$$

where μ_b = the mean plastic moment strength of beam of top floor, μ_{bij} = the mean moment strength of beam below top floor, μ_{csl} = the mean plastic moment strength of exterior column, and μ_{cl} = the mean moment strength of interior column. In all the computations within the present paper the mean plastic moment strength of top beams of frame structures are assumed to be 104.1 $kN\cdot m$. Since the earthquake load is inverse triangularly distributed, the mean value of load applied on each floor of the structure is in proportion, namely,

$$\mu_{pj} = j\mu_p \tag{2}$$

where μ_{pj} = the mean value of load acting on the jth floor, and μ_p = the mean value of load acting on the first floor, that is μ_p=μ_{p1}.

All the stochastic variables are assumed to obey a lognormal distribution, and the coefficients of variation are defined by V_1 = 0.1 for member strength and V_2 = 0.8 for earthquake load[12].

Assume that C_{of} = 1.3, using stochastic limit analysis[11), 13)], the first nine most likely failure modes are obtained and defined as Mode 1 ~ 9 in sequence according to their reliability indices by first order reliability method[14] (FORM), as shown in Fig. 2.

Obviously, each of these failure modes has different significance to structural design.

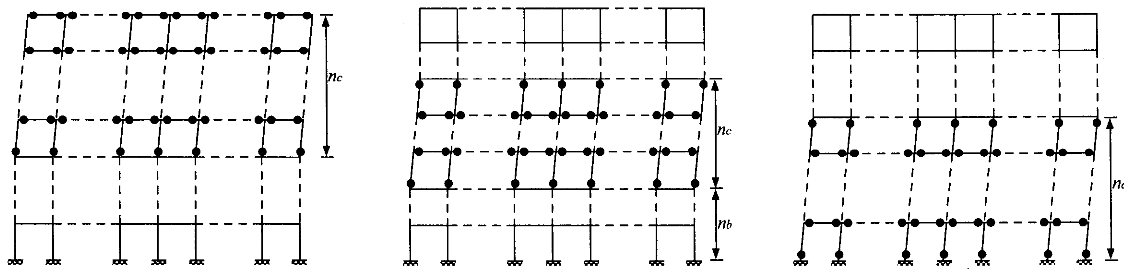
1). Mode 1: Mode 1 is the strict entire beam-hinging mode, in which plastic hinges develop in all the beam edges and the column bases of the ground floor. This mode is the preferable failure mode due to its large capability to absorb earthquake energy before collapse. One basic concept of probabilistic design it to assure this mode form more likely than other modes, thus the occurrence probability of this mode should be higher than those of other failure modes.

2). Mode 2, 5 and 7: These modes are similar to mode 1. Even though some hinges occurred in the top edges of columns of top floor and they are considered less desirable, basically the yielding of the beams, except for some in the top story, is prior to the yielding of columns, so these modes are named nearly beam-hinging failure mode.

3). Mode 3 and 6: The hinges developed in the columns of the middle stories. Since some hinges develop in the columns of middle stories, they do not belong to the beam-hinging pattern, but are still categorized within the entire collapse pattern.

4). Mode 4, 8 and 9: The hinges develop at all the top edges and bases of columns in one or several continuous stories, and the structure collapses as story mechanism.

Among all the 4 types of failure modes presented above, mode 1 is undoubtedly the preferred mode, and generally other modes are considered as undesirable modes. In practical seismic design, however, it is difficult to ensure the structure collapse according to the entire beam-hinging pattern, so the criterion generally becomes to avoid story mechanisms, and therefore the failure modes of No. 2 and No. 3 types are allowable, even though they are not the preferred pattern.



(a) Upper Collapse Type (b) Lower Collapse Type (c) Middle Collapse Type

Fig. 3. Story Collapse Types

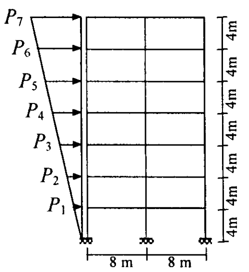


Fig. 4. 7-story-2-bay Frame for Example

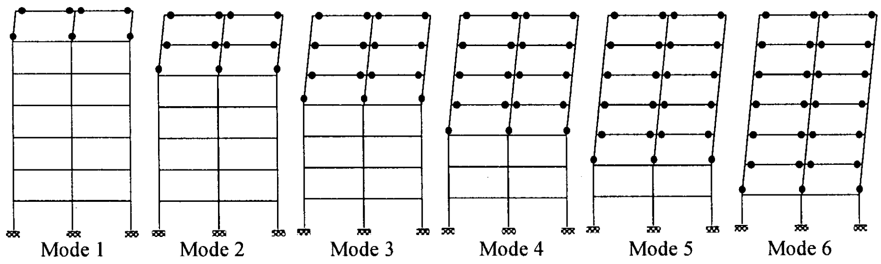


Fig. 5. Upper Collapse Modes for 7-story-2-bay Frame

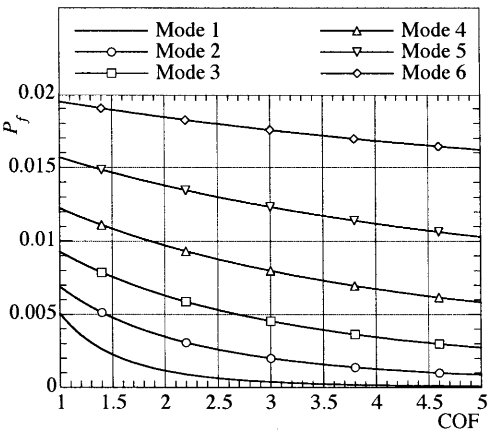


Fig. 6. P_f-C_{of} Curves for Upper Collapse Modes

Therefore, the failure modes can be defined generally in three types:

(1). Preferable mode, i.e., the strictly entire beam-hinging pattern, mode 1 in Fig. 2;

(2). Allowable modes, including the nearly entire beam-hinging pattern and entire collapse pattern, mode 2, 3, 5, 6, and 7 in Fig. 2;

(3). Unallowable modes, i.e., the story mechanism, mode 4, 8, 9 in Fig. 2.

In order to avoid story mechanisms probabilistically, it is necessary to investigate the probabilities and occurrence orders of story mechanisms.

4. PROBABILISTIC INVESTIGATION ON STORY COLLAPSE MODES

4.1 Classification of Story Mechanisms

When the investigation is focused on story mechanisms, the number of failure modes will be greatly reduced. However, there are still generally 2^n-1 story mechanisms for an n-story structure, including the entire beam-hinging mode. It is obviously still troublesome to investigate all these failure modes. In order to reduce the number of failure modes that should be exhibited in structural design or analysis, the occurrence orders of these modes are investigated here. In this paper the story failure modes are classified into three patterns: upper collapse pattern, middle collapse pattern and lower collapse pattern, as shown in Fig. 3. Upper collapse pattern is characterized by the continuous collapsed stories from the top story; lower collapse pattern is characterized by the continuous collapse of stories from the first story; middle collapse pattern is with collapse of stories in the middle structure, and the stories at the top and bottom remain unbroken. Since the degree of prevalence of failure modes combined with any of the collapse types mentioned above are obviously less than that of each single type, the combination of the three collapse types are not considered in the investigation. Then, there are $n(n+1)/2$ story mechanisms for an n-story structure need to be investigated. The performance functions of all the modes belonging to the same type can be unified by one formula. Taking a 2-bay-7-story frame structure shown in Fig. 4 as example, the failure probabilities of these types are computed and analyzed.

4.2 Determination of Load Level

As mentioned above, the entire beam-hinging mode is the preferable failure pattern of frame structure, and its failure probability can be computed by FORM. This failure probability or the corresponding reliability index reflects the safety of structure if the entire beam-hinging pattern is the most likely failure mode. To ensure meaningful comparison of the failure probabilities of story mechanisms, it is required that all the frame structures are designed under the same reliability level. Under this assumption, according to a designed reliability index of the preferable entire beam-hinging failure pattern, 41 mean values of the earthquake load can be computed out by changing the COF from 1.0 to 5.0 by step of 0.1, and the loads are then applied to compute the failure probability of each story mechanism. It has been indicated that the load level is proportional to the COF[15)]. In this study, the earthquake loads, which lead to an invariable reliability index of entire beam-hinging failure pattern β_T= 2.0 in whole the COF region, are used in the analysis.

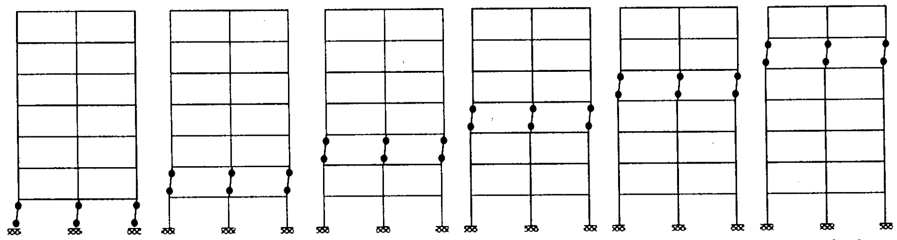
4.3 Investigation on Upper Story Mechanisms

For a 7-story structure, 6 upper collapse modes are likely to develop. Figure 5 shows the distributions of plastic hinges in the upper collapse modes. Using FORM, the computed P_f-C_{of} curves are shown in Fig. 6. From the numerical results one can see that for upper collapse type with the increase of the number of collapsed stories, the failure probability becomes larger.

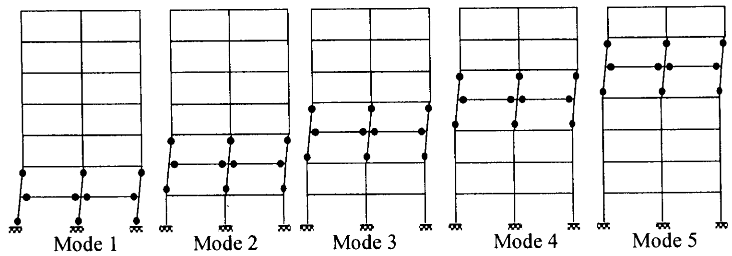
According to the collapse form showed in Fig. 3(a), the corresponding performance function can be established based on the principle of virtual work as:

$$G(\mathbf{X}) = 2\sum_{i=1}^{m} M_{bni} + 2\sum_{j=n-n_c+1}^{n-1}\sum_{i=1}^{m} M_{bij} + \sum_{l=1}^{2} M_{csl} + \sum_{l=1}^{m-1} M_{cl} - \sum_{j=n-n_c+1}^{n} (j+n_c-n)hP_j \tag{3}$$

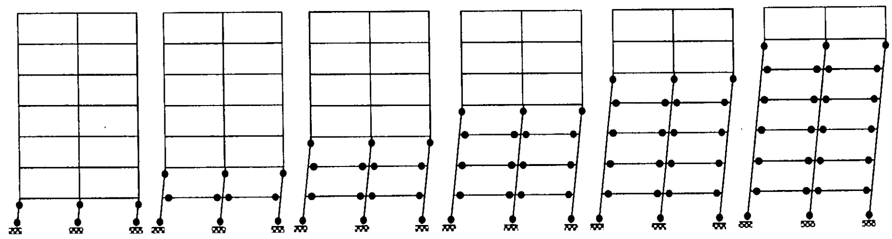
where M_{bni} is the plastic moment strength of top beams; M_{bij} is the plastic moment strength of beams below top floor; M_{csl} is the plastic moment strength of exterior columns, M_{cl} is the plastic moment strength of interior



(a) Middle Collapse Modes for n_c=1



(b) Middle Collapse Modes for n_c=2



(c) Lower Collapse Modes

Fig. 7. Middle and Lower Collapse Modes for 7-story-2-bay Frame

columns. P_j is the load acting on the jth story of the structure. n and m are the number of stories and spans, respectively. h is story height, and n_c is the number of collapsed stories.

The second moment reliability index of upper collapse type can be given by (see Appendix 1),

$$\beta_{SM-U} \approx \frac{2m(2n_c-1+C_{of})\mu_b - \mu_p h \sum_{j=n-n_c+1}^{n} j(j+n_c-n)}{V_2\mu_p h\sqrt{\sum_{j=n-n_c+1}^{n} j^2(j+n_c-n)^2}} \quad (4)$$

In order to investigate the monotony of the second moment reliability index, rewrite Eq. (4) as:

$$\beta_{SM-U} = A\sqrt{f_1(n_c)} + B\sqrt{f_2(n_c)} - \frac{1}{V_2}\sqrt{f_3(n_c)} \quad (5)$$

where A and B are two positive polynomials independent of n_c:

$$A = \frac{2m\mu_b(C_{of}-1)}{V_2\mu_p h}, \quad B = \frac{4m\mu_b}{V_2\mu_p h} \quad (6)$$

and,

$$f_1(n_c) = [\sum_{j=n-n_c+1}^{n} j^2(j+n_c-n)^2]^{-1} \quad (7.a)$$

$$f_2(n_c) = \frac{n_c^2}{\sum_{j=n-n_c+1}^{n} j^2(j+n_c-n)^2} \quad (7.b)$$

$$f_3(n_c) = \frac{1}{\sum_{j=n-n_c+1}^{n} j^2(j+n_c-n)^2}\left[\sum_{j=n-n_c+1}^{n} j(j+n_c-n)\right]^2 \quad (7.c)$$

f_1, f_2 and f_3 are three new functions of n_c. Once the monotony of f_1, f_2 and f_3 to n_c is known, the monotony of β_{SM-U} will also be known. It has been demonstrated that f_1 and f_2 decrease and f_3 increases with the increase of n_c (Appendix 2), so one can know that β_{SM-U} decreases monotonically with the increase of n_c. It means that for an n-story frame structure the upper collapse mode with n-1 collapsed stories, has the minimum reliability index, i.e., it is the most likely one within all the upper collapse modes.

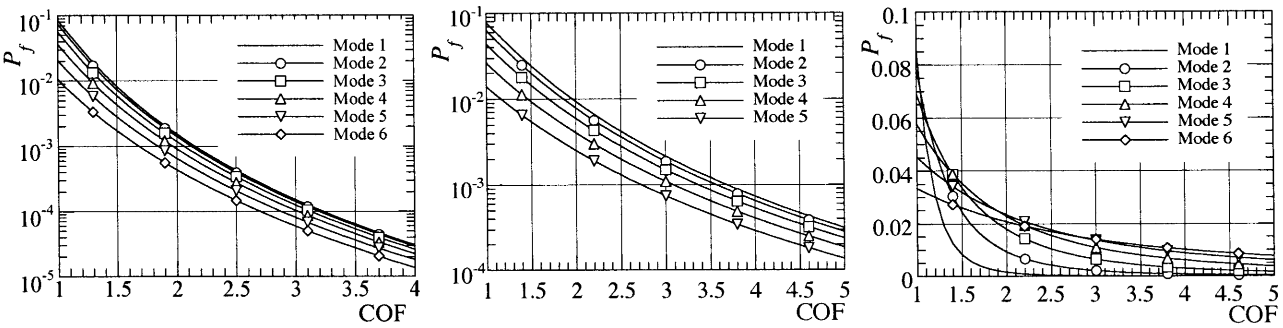
4.4 Investigation on Middle Story Mechanisms

Middle story mechanism develops if the plastic hinges form in the middle stories while the stories at top and bottom of structure remain intact, as shown in Fig. 3(b). The corresponding performance function can be given by:

$$G(\mathbf{X}) = 2\sum_{j=1}^{n_c-1}\sum_{i=1}^{m} M_{bij} + \sum_{l=1}^{4} M_{csl} + \sum_{l=1}^{2m-2} M_{cl} - \sum_{j=1}^{n_c} jhP_{j+n_b} - \sum_{j=n_c+1}^{n-n_b} n_c hP_{j+n_b} \quad (8)$$

where n_b is the number of unbroken stories at bottom.

For the 7-story-2-span structure, the middle collapse modes are shown in Figs. 7(a) and 7(b) for n_c = 1 and n_c = 2, respectively. The P_f-C_{of} curves computed by FORM are depicted in Fig. 8 (a) for n_c = 1 and in Fig. 8 (b) for



(a) Middle Collapse Modes: n_c=1 (b) Middle Collapse Modes: n_c=2 (c) Lower Collapse Modes

Fig. 8. P_f-C_{of} Curves for Middle and Lower Collapse Modes

$n_c = 2$.

From both Fig. 8 (a) and Fig. 8 (b), one can see that the larger the n_b, the smaller the P_f, i.e., the failure probability of middle collapse mode decreases with the shifting of the plastic hinges from lower stories to the upper stories. When n_b = 0, the middle collapse mode becomes lower collapse mode. One can see from Figs. 8(a) & 8(b) that the failure probability of lower collapse mode is always larger than that of middle collapse mode with the same number of collapsed stories. For other cases of n_c, the same tendency can also be found from the P_f-C_{of} curves. Therefore, the middle collapse modes are not the most likely failure mode.

In order to understand the relationship between the lower collapse types and middle collapse types, consider the second moment reliability index of the middle collapse type given by (Appendix 3):

$$\beta_{SM-M} \approx \frac{4m\mu_b(n_c-1+C_{of})-\mu_p h\left[\sum_{j=1}^{n_c} j(j+n_b)+n_c\sum_{j=n_c+1}^{n-n_b}(j+n_b)\right]}{V_2\mu_p h\sqrt{\sum_{j=1}^{n_c} j^2(j+n_b)^2+n_c^2\sum_{j=n_c+1}^{n-n_b}(j+n_b)^2}} \quad (9)$$

In order to demonstrate the monotony of the second reliability index of the middle collapse type with n_b, the parameters excluding n_b are all assumed to be invariable. Rewrite Eq. (9) as:

$$\beta_{SM-M} = \frac{4m\mu_b(n_c-1+C_{of})-\mu_p hC}{V_2\mu_p h\sqrt{D}} \quad (10)$$

in which C and D are the function of n_b given by:

$$C(n_b) = \sum_{j=1}^{n_c} j(j+n_b)+n_c\sum_{j=n_c+1}^{n-n_b}(j+n_b) \quad (11.a)$$

$$D(n_b) = \sum_{j=1}^{n_c} j^2(j+n_b)^2+n_c^2\sum_{j=n_c+1}^{n-n_b}(j+n_b)^2 \quad (11.b)$$

It is known that both $C(n_b)$ and $D(n_b)$ decrease with the increase of n_b (Appendix 4), and consequently the second moment reliability index of the middle collapse type increases with the increase of the number of unbroken stories at bottom.

4.5 Investigation on Lower Story Mechanisms

As described above, the lower collapse type can be considered as a special case of middle collapse modes when n_b = 0, and the performance function can be obtained by assuming n_b in Eq. (8) is 0 as:

$$G(\mathbf{X}) = 2\sum_{j=1}^{n_c-1}\sum_{i=1}^{m} M_{bij} + \sum_{l=1}^{4} M_{csl} + \sum_{l=1}^{2m-2} M_{cl} - \sum_{j=1}^{n_c} jhP_j - \sum_{j=n_c+1}^{n} n_c hP_j \quad (12)$$

The parameters are the same as in the upper collapse type.

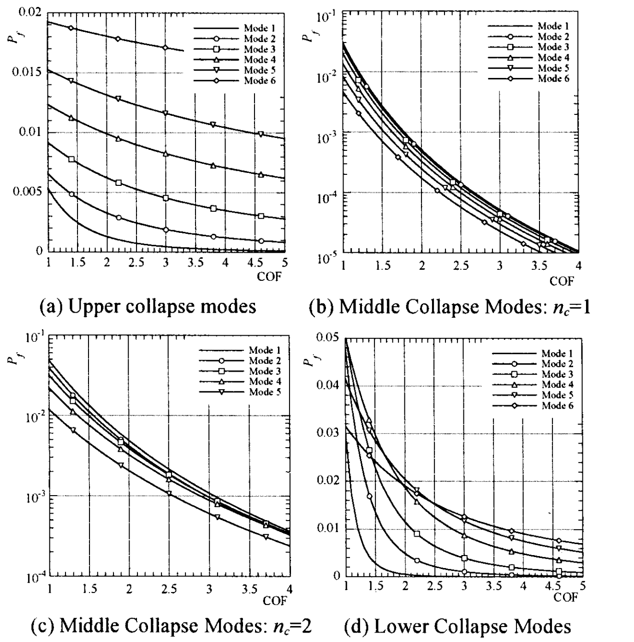
For the lower collapse type of the 7-story structure, the 6 potential lower collapse modes are shown in Fig. 7(c). The computed P_f-C_{of} curves are compared in Fig. 8(c), from which it can be found that each mode has a special COF region where the failure probability of this mode is relatively the greatest. That is to say, every lower collapse mode has the likelihood to form first at a specific COF level. The variation of COF has a great influence on the occurrence order of lower collapse modes. It can be observed from Fig. 8(c) that the increase of COF causes relatively high possibility of the developing of plastic hinges in more stories, namely the structure designed with high COF tends to collapse in a pattern close to the entire beam-hinging mode for the great weak-beam-strong-column effect. This result agrees with that obtained in the research by Ogawa[7)].

To get an overall understanding on the most likely story mechanisms, it is necessary to have a comparison between the upper collapse pattern and lower collapse pattern even though it is difficult to give a clear demonstration on their relationships. From Fig. 6 and Fig. 8(c) one can see that the maximum probabilities of all the upper collapse modes ranges about from 0.015 to 0.02, and compared with the probabilities of all the lower collapse modes it is relatively large for high-level column over-designed fames but small for low-level column over-designed frames. Because both the upper collapse mode and the lower collapse modes are likely to be the dominant story failure mode, it is suggested to take into account all the lower collapse modes and the upper collapse mode with most failure stories together when assessing structural reliability.

4.6 Investigation on Frames with Non-uniformly Distributed Strengths

The analysis presented above is concerned about frames with uniformly distributed strengths over their height, in which the column strengths in each floor are identical and the COF values in each beam-column node are the same. To understand whether the frames with non-uniformly distributed strengths obey the rules obtained from frame with uniform strengths, it is required to conduct a numerical investigation on the story mechanisms of the frame with ununiformly distributed strength.

Generally, the columns in lower stories of frame structures are designed stronger than those in upper stories because they carry relatively larger load, and that means the larger COF values in the beam-column nodes of lower stories. Taking the 2-span-7-story frame shown in Fig. 3 as example and assuming that 1.1 times of the exterior column strength of top floor are designed for exterior columns in 4~6 floor, and 1.2 times for the exterior



(a) Upper collapse modes (b) Middle Collapse Modes: n_c=1

(c) Middle Collapse Modes: n_c=2 (d) Lower Collapse Modes

Fig. 9. P_f-C_{of} Curves for Strength Non-uniformly Designed Frame

columns in 1~3 floor, and the other parameters are the same as used in the previous analysis, the P_f-C_{of} curves can be obtained as shown in Fig. 9, where the COF value is that of beam-column node in top story, the minimum one of all the COF values of frame. From Fig. 9 basically the same results as obtained in the analysis of the frame with uniform distributed strengths can be observed. Even though it is difficult to draw a definite conclusion theoretically from the reliability indices, the results of this example indicate that the rules of story mechanisms of frame with uniform strengths are generally applicable for those of the frame with slightly non-uniformly distributed strengths.

5. CONCLUSIONS

The preferable and undesirable failure modes for aseismic design of frame structures are analyzed, and the failure probabilities of the story mechanisms of frame structures are investigated probabilistically through both numerical and theoretical analysis. It was found that:

(1) Regardless of the COF, for the upper story mechanisms the larger the number of the collapse stories, the higher the occurrence probability;

(2) For the middle story mechanisms with an identical number of collapse stories, the plastic hinges are more likely to develop in lower stories of structure;

(3) For the story mechanisms with the same number of failure stories, the probability of the lower collapse mode is always higher than that of the middle collapse mode if all the beam-column nodes are designed with the same COF;

(4) The probabilistic orders of lower story mechanisms are affected notably by the COF values of frames. For low-level column over-designed frames, the plastic hinges tend to develop in less stories; for high-level column over-designed frames, the plastic hinges tend to develop in more stories.

Based on these results, for a specific n-story frame structure considered in design, the most likely story mechanisms can be identified as the n-1 lower story mechanisms and the upper story mechanism with n-1 failure stories.

It should be noted that the investigation was conducted under some restrictive assumptions described in the paper. Some other factors that are not considered in this study, such as second-order effects and axial deformation, type of ground motion and dynamic response, distribution type of random variables, definition of the beam-hinging pattern, and correlation among member strengths, may be important, and further researches involving these factors are needed.

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APPENDIX 1: The Reliability Index of Upper Collapse Type

In order to investigate the monotony of occurrence probability of the upper collapse type, rewrite the performance function as:

$$G(\mathbf{X}) = x_m - x_p \tag{13}$$

where,

$$x_m = 2\sum_{i=1}^{m} M_{bni} + 2\sum_{j=n-n_c+1}^{n-1}\sum_{i=1}^{m} M_{bij} + \sum_{l=1}^{2} M_{csl} + \sum_{l=1}^{m-1} M_{cl} \tag{14.a}$$

$$x_p = \sum_{j=n-n_c+1}^{n} (j+n_c-n)hP_j \tag{14.b}$$

It is assumed that the structure was designed with one COF in all the column-beam nodes, and the mean values and coefficients of variation of random variables are defined as:

$$\mu_{bni} = \mu_b,\quad \mu_{bij} = 2\mu_b$$

$$\mu_{csl} = C_{of}\mu_b,\quad \mu_{cl} = 2C_{of}\mu_b,\quad \mu_{pj} = j\mu_p \tag{15}$$

$$V_{bni} = V_{bij} = V_{cl} = V_{csl} = V_1,\quad V_{pj} = V_2 \tag{16}$$

Then the mean value and standard variation of x_m and x_p are given as following:

$$\mu_{x_m} = 2m(2n_c - 1 + C_{of})\mu_b \tag{17.a}$$

$$\mu_{xp} = \mu_p h \sum_{j=n-n_c+1}^{n} (j + n_c - n) \tag{17.b}$$

$$\sigma_{x_m} = V_1 \mu_b \sqrt{2\left[(2m-1)C_{of}^2 + 8m(n_c - 1) + 2\right]} \tag{17.c}$$

$$\sigma_{x_p} = V_2 \mu_p h \sqrt{\sum_{j=n-n_c+1}^{n} j^2 (j + n_c - n)^2} \tag{17.d}$$

The mean value and standard variation of performance function G are thus:

$$\mu_G = 2m(2n_c - 1 + C_{of})\mu_b - \mu_p h \sum_{j=n-n_c+1}^{n} (j + n_c - n) \tag{18.a}$$

$$\sigma_G^2 = 2V_1^2 \mu_b^2\left[(2m-1)C_{of}^2 + 8m(n_c - 1) + 2\right] + V_2^2 \mu_p^2 h^2 \sum_{j=n-n_c+1}^{n} j^2 (j + n_c - n)^2 \tag{18.b}$$

Meanwhile since $V_1^2 << V_2^2$, σ_G can be approximately given as:

$$\sigma_G \approx \sigma_{x_p} = V_2 \mu_p h \left[\sum_{j=n-n_c+1}^{n} j^2 (j + n_c - n)^2\right]^{1/2} \tag{19}$$

Then the approximate expression of the second reliability index is obtained as Eq. (4).

APPENDIX 2: The Monotony of f_1, f_2 and f_3 to n_c

Among the three functions in Eq. (7), f_1 obviously decreases monotonically with the increase of n_c. f_2 can be expanded as:

$$f_2 = \frac{30n_c}{(1+n_c)(-1+5n^2+n_c+5nn_c+10n^2n_c-n_c^2-5nn_c^2+n_c^3)} \tag{20}$$

Then to differentiate f_2, it is obtained as:

$$\frac{df_2}{dn_c} = \frac{-30(1-5n^2+10n^2n_c^2-10nn_c^2+3n_c^4)}{(1+n_c)^2(-1+5n^2+n_c+5nn_c+10n^2n_c-n_c^2-5nn_c^2+n_c^3)^2} \tag{21}$$

Since total story number $n \geq 2$, one can easily understand that:

$$1-5n^2+10n^2n_c^2-10nn_c^2+3n_c^4 = 5n[2n_c^2(n-1)-n]+3n_c^2+1 > 0 \tag{22}$$

Therefore $f_2'(n_c) < 0$, and it means f_2 decreases monotonically with the increase of n_c.

Similarly expanding f_3 as:

$$f_3 = \frac{5n_c(1+n_c)(1-n_c+3n)^2}{6(-1+5n^2+n_c+5nn_c+10n^2n_c-n_c^2-5nn_c^2+n_c^3)} \tag{23}$$

then differentiate f_3 as:

$$\begin{aligned}\frac{df_3}{dn_c} = \frac{5}{6}[&(11n^2n_c^4+n_c^6-10nn_c^5)+10n^2n_c^2(3n-2n_c)^2 \\ &+2n_c(45n^4-39n^2n_c^2+13nn_c^3-n_c^4)+(34n^4+23n^2n_c^2-34nn_c^3+5n_c^4) \\ &+2(15n^3+12nn_c^2-14n^2n_c-4n_c^3)+(11n^4-4n^2+3n_c^2-6n+2n_c-1)] \\ &/(-1+n_c+5n^2+5nn_c+10n^2n_c-n_c^2-5nn_c^2+n_c^3)^2\end{aligned} \tag{24}$$

In Eq. (24), all the items in the square bracket are arranged for the easy understanding that the polynomials in parentheses each is larger than zero, and thus it is known that $f_3'(n_c) > 0$. Namely f_3 increases monotonically with n_c.

APPENDIX 3: The Second Moment Reliability Index of the Middle Collapse Type

Similar to Appendix 1, rewrite the performance function Eq. (8) as Eq. (13), but here:

$$x_m = 2\sum_{j=1}^{n_c-1}\sum_{i=1}^{m} M_{bij} + \sum_{l=1}^{4} M_{csl} + \sum_{l=1}^{2m-2} M_{cl} \tag{25.a}$$

$$x_p = \sum_{j=1}^{n_c} jhP_{j+n_b} + \sum_{j=n_c+1}^{n-n_b} n_c hP_{j+n_b} \tag{25.b}$$

According to Eqs. (15) and (16), the mean value and standard variation of x_m and x_p are given by:

$$\mu_{x_m} = 4m\mu_b(n_c - 1 + C_{of}) \tag{26.a}$$

$$\mu_{x_p} = \mu_p h\left[\sum_{j=1}^{n_c} j(j+n_b) + n_c \sum_{j=n_c+1}^{n-n_b} (j+n_b)\right] \tag{26.b}$$

$$\sigma_{x_m} = V_1 \mu_b \sqrt{2m(n_c-1) + C_{of}^2 + (2m-2)C_{of}^2} \tag{26.c}$$

$$\sigma_{x_p} = V_2 \mu_p h \sqrt{\sum_{j=1}^{n_c} j^2 (j+n_b)^2 + n_c^2 \sum_{j=n_c+1}^{n-n_b} (j+n_b)^2} \tag{26.d}$$

Then the mean value and standard variation of performance function G are obtained as:

$$\mu_G = 4m\mu_b(n_c - 1 + C_{of}) - \mu_p h\left[\sum_{j=1}^{n_c} j(j+n_b) + n_c \sum_{j=n_c+1}^{n-n_b} (j+n_b)\right] \tag{27.a}$$

$$\sigma_G^2 = V_1^2 \mu_b [2m(n_c-1) + (2m-1)C_{of}^2] + V_2^2 \mu_p^2 h^2 \left[\sum_{j=1}^{n_c} j^2 (j+n_b)^2 + n_c^2 \sum_{j=n_c+1}^{n-n_b} (j+n_b)^2\right] \tag{27.b}$$

Since $V_1^2 << V_2^2$, $\beta_{SM\text{-}M}$ can be approximately expressed by Eq. (9).

APPENDIX 4: The Monotony of $C(n_b)$ and $D(n_b)$ to n_b

Obviously, $C > 0$ and $D > 0$. If using $C(n_b+1)$ to subtract $C(n_b)$, then:

$$\begin{aligned}&C(n_b+1) - C(n_b) \\ &= \left[\sum_{j=1}^{n_c} j(j+n_b+1) + n_c \sum_{j=n_c+1}^{n-n_b-1} (j+n_b+1)\right] - \left[\sum_{j=1}^{n_c} j(j+n_b) + n_c \sum_{j=n_c+1}^{n-n_b} (j+n_b)\right] \\ &= -\frac{1}{2}(n_c - 1 + 2n - 2n_b) < 0\end{aligned} \tag{28}$$

Here n_c obeys that $n_c \geq 1$. It is indicated from the obtained result that $C(n_b)$ has the monotony decreasing with n_b. Similarly subtract $D(n_b)$ from $D(n_b+1)$, it is found that:

$$\begin{aligned}&D(n_b+1) - D(n_b) = \left[\sum_{j=1}^{n_c} j^2 (j+n_b+1)^2 + n_c^2 \sum_{j=n_c+1}^{n-n_b-1} (j+n_b+1)^2\right] \\ &-\left[\sum_{j=1}^{n_c} j^2 (j+n_b)^2 + n_c^2 \sum_{j=n_c+1}^{n-n_b} (j+n_b)^2\right] \\ &= -\frac{1}{6}[3n_c^3 - 1 + 6n_c n_b(1+n_b) + 4n_c^2(1+2n_b) - 2n_b] < 0\end{aligned} \tag{29}$$

Then one can easily understand that $D(n_b)$ also decreases monotonically with n_b.

和文要約

1. はじめに

特定の崩壊機構で骨組を崩壊させることは、構造設計上で合理的と考えられる。耐震設計では、層崩壊機構は望ましくない崩壊機構であるので、出現しやすい層崩壊機構を考察することは重要である。本論では骨組の層崩壊機構を上部層崩壊、中間層崩壊と下部層崩壊の三パータンに分類し、各層崩壊パータンの生起確率を求め、そして最も出現しやすい層崩壊機構を確率論的に考察することを目的とする。

2. 基本仮定

(1) ラーメンを構成する部材は、完全剛塑性挙動するものとする。断面の破壊はこの断面に塑性ヒンジが生じることを意味する。

(2) 骨組の各パラメータの中に、部材強度は唯一の確率変数である。

(3) 幾何学的二次効果と軸力の影響を無視する。

(4) 地震荷重は逆三角形分布の静的荷重とする。

3. 崩壊レベルの考察

図 1 に示す各節点が同じ COF(Column Over-design Factor, 柱梁耐力比)で設計された 3 層 3 スパン骨組に対して、C_{of}=1.3 のとき、確率極限解析法で求められた 9 個の出現しやすい崩壊機構が図 2 に示された。中では：

(1) モード 1：厳密な梁降伏先行型全体崩壊機構。この崩壊機構では、柱部分の一層柱脚以外には塑性ヒンジが起こらない。このような崩壊型は十分なエネルギー吸収能力を持っているので、耐震設計上では望ましい崩壊機構とされている。

(2) モード 2、5 と 7：モード 1 とほぼ同じで、最上層のある柱頭を除いて、基本的に柱より梁の降伏が先に発生すると言える。

(3) モード 3 と 6：中間層の柱に塑性ヒンジが発生するので、梁降伏型ではないが、全体崩壊機構と見なせる。

(4) モード 4、8 と 9：同一層のすべての柱に塑性ヒンジが発生しているので、層崩壊機構が形成する。

この四種類の崩壊機構の中で、確かにモード 1 は最も望ましい崩壊機構である。しかし、実際の建物が梁降伏全体崩壊機構によって崩壊するように設計されても、必ずしも設計された崩壊機構で崩壊するとは限らない。耐震設計では原則的に層崩壊機構を回避するので、崩壊機構を以下のように分類する。

(1) 望ましい崩壊機構：モード 1；

(2) 許容する崩壊機構：モード 2、3、5、6 と 7；

(3) 許容しない崩壊機構：モード 4、8 と 9。

4. 層崩壊機構生起確率の考察

梁降伏全体崩壊機構を含めて、n 層のラーメン骨組の可能な層崩壊機構は 2^n-1 個である。構造設計と解析上で検討する必要な崩壊機構を減少させるために、層崩壊機構の生起確率を考察することが必要となる。骨組の層崩壊機構が上部層崩壊機構、中間層崩壊機構と下部層崩壊機構に分類されて、それぞれが骨組の上部、中間部と下部に連続的な崩壊層が形成する崩壊機構である（図 3）。実際にこの三種類の組み合わせの層崩壊機構も可能であるが、この三種類の層崩壊機構の生起確率と比べて、組み合わせパータンの生起確率はかなり小さいので、考慮しなくてもよい。 従って、検討する層崩壊機構の数が $n(n+1)/2$ になる。図 4 に示す 7 層 2 スパン骨組の数値解析と共に、各層崩壊機構の生起確率の順番を検討する。

4.1 上部層崩壊機構

図 5 に 6 個の上部層崩壊機構を示す。FORM で信頼性解析を行って、求めてきた P_f-C_{of} 関係曲線が図 6 に示す。図 6 から崩壊層数の増加につれて上部層崩壊機構の生起確率が大きくなることが分かった。

各崩壊モードの生起確率の関係を調べるため、仮想仕事の原理により上部層崩壊機構の限界状態関数は式（3）のようになる。それに対応する二次モーメント信頼性指標が式（4）になる。式（4）を式（5）のように書き直すことができる。中には、係数 A、B が式（6）に、$f_1(n_c)$、$f_2(n_c)$ と $f_3(n_c)$ がそれぞれ式（7.a）、(7.b）と（7.c）に定義される。付録 2 に証明した n_c の増加と共に $f_1(n_c)$、$f_2(n_c)$ が減少、$f_3(n_c)$ が増加することによって、上部層崩壊機構の二次モーメント信頼性指標が減少することが分かる。

4.2 中間層崩壊機構

7 層の骨組に対して、崩壊層数 n_c=1 と n_c=2 の場合の崩壊モードがそれぞれ図 7（a）と図 7（b）に示す、得られた曲線がそれぞれ図 8（a）と図 8（b）に示されている。図 8（a）と図 8（b）によると、骨組下部の崩壊しない層の数 n_b の増加と共に、崩壊モードの生起確率が減少する。n_b = 0 の時、中間層崩壊機構が下部層崩壊機構になる。

中間層崩壊機構の限界状態関数が式（8）で表される。二次モーメント信頼性指標が式（9）で表され、n_b の関数 $C(n_b)$、$D(n_b)$ を式（11）のように定義すると、二次モーメント信頼性指標が式（10）になる。付録 4 により、n_b の増加と共に、$C(n_b)$ と $D(n_b)$ が減少する。従って、二次モーメント信頼性指標が n_b の増加によって減少する。つまり、同じ崩壊層数をもつ下部層崩壊モードは中間層崩壊モードより生起確率が高い。

4.3 下部層崩壊機構

前節で述べたように、中間層崩壊機構が n_b = 0 のとき下部崩壊機構になるので、式（8）の n_b を 0 とすると、下部層崩壊機構の限界状態関数が式（12）のように得られた。

7 層骨組の 6 個の下部層崩壊モードが図 7 (c) に表される。図 8 (c) に表される P_f-C_{of} 曲線を見ると、COF の値によって、すべての下部層崩壊機構の生起確率が最大になる可能性があり、設計するとき全部の下部層崩壊機構を検討する必要がある。

5. 結論

数値解析および理論証明により、最も出現しやすい層崩壊機構を確率論的に考察した。n 層骨組の $n(n+1)/2$ 個の層崩壊機構の中に、崩壊層数が n-1 の上部層崩壊機構とすべての下部層崩壊機構は最も出現しやすい層崩壊機構である。

（付録 1：上部層崩壊機構の信頼性指標；付録 2： n_c に対する f_1、f_2 と f_3 の単調性；付録 3：中間層崩壊機構の信頼性指標；付録 4： n_b に対する $C(n_b)$ と $D(n_b)$ の単調性）

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