System Reliability Evaluation of Ductile Frame Structures

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Abstract: The system reliability evaluation of ductile frame structures presents two difficulties. One is the identification of significant failure modes; the other is the computation of overall failure probabilities contributed from these significant modes. In this paper, a practical procedure is developed, in which a failure mode independent performance function is defined using load factor obtained by limit analysis, and a response surface approach is used to approximate the performance function. With the performance function of explicit second-degree polynomial, the failure probability can be readily obtained using First and Second Order Reliability Method (FORM/SORM). Several examples are investigated, and it is found that the obtained response surface is a good approximation of the inner connotative surface of the limit state surfaces of the structural system. The proposed procedure has good efficiency and enough accuracy for system reliability evaluation of ductile frame structures. The difficulty in both failure mode identification and failure probability computation can be avoided by using the proposed procedure.

INTRODUCTION

The evaluation of system reliability for ductile frame structures has been an active area of research for nearly 30 years. During this period, efficient procedures have been developed for reliability evaluation of individual limit state. The computation of system reliability, which is affected by many interacting limit states, still presents considerable difficulties and expenses. The search for efficient computational procedures to estimate system reliability has resulted in several approaches, such as the failure modes approach and direct Monte Carlo simulation. In this paper, a practical procedure is proposed, in which a failure mode independent performance function is defined and a response surface approach is used to approximate the performance function. With the performance function of explicit second-degree polynomial, the failure probability can be readily obtained using First and Second Order Reliability Method (FORM/SORM).

SYSTEM RELIABILITY OF DUCTILE FRAME STRUCTURES

For ductile frame structures considered in this study, several commonly used assumptions are applied.

1. Elasto-plastic frame structures are considered. The failure of a section means the imposition of a hinge and an artificial moment at this section.
2. Structural uncertainties are represented by considering only moment capacities as random variables.
3. Geometrical second-order and shear effects are neglected. The effects of axial forces on the reduction of moment capacities are also neglected.

Based on the upper-bound theorem of plasticity (Livesley 1976), failure of a ductile frame structure is defined as the formation of a kinematically admissible mechanism due to the formation of plastic hinges at a certain number of sections.

Since a structural system may fail in different modes, the general definition of performance function for ductile frame structures is the minimum value of the performance functions corresponding to all the potential failure modes as shown in (1).

$$G(\mathbf{M}, \mathbf{P}) = \min[G_1(\mathbf{M}, \mathbf{P}), G_2(\mathbf{M}, \mathbf{P}), \ldots, G_n(\mathbf{M}, \mathbf{P})] \quad (1)$$

where $\mathbf{M}$ = vector of moment capacities of the structures; and $\mathbf{P}$ = load vector. $G_i(\mathbf{M}, \mathbf{P})$ is the performance function corresponding to ith failure mode.

To obtain the performance function defined by (1), the potential failure modes should be identified. However, there is generally an astronomically large number of potential failure paths in large-scale structures, and in most cases only a small fraction of them contribute significantly to the overall failure probability. These may be referred to as significant failure sequences, and the estimation of system failure probability based on these sequences is expected to be close to the true answer.

Many types of systematic component failure generation procedures have been developed to identify this set of failure paths with relatively higher failure probability, such as enumeration-type techniques (Thoft-Christensen and Murotsu 1986). Among the enumeration techniques, the truncated enumeration method (Melchers and Tang 1985; Nafday 1987; Xiao and Mahadeven 1994) provided a systematic, rational derivation and generalized earlier methods in this category such as the branch and bound method (Murotsu et al. 1984) and incremental load method (Moses 1982). However, these methods result in a large number of failure paths and structural reanalysis, which makes the computation very expensive even for small-size structures. Other methods for identifying the significant failure mode include the stable configuration approach (Bennett and Ang 1987). Mathematical programming techniques (Zimmerman et al. 1993; Ohi 1991) can give the likely failure modes with less computation time, but the guarantee against the losing of significant modes is found to be difficult.

The computation of failure probability is difficult even if the potential failure modes have been known, because of the large number of the failure modes, the difficulty in obtaining the sensitivity of performance function in (1), and the correlation among the failure modes. Many approximate methods such as bounding techniques [e.g., Cornell (1966); Schueler and Stix (1987)], probabilistic network evaluation technique (PNET) (Ang and Ma 1981), high-order moment standardization technique (Ono et al. 1990), and the point estimation method (Idota et al. 1991) have been developed. There is still no method that satisfies both the requirements of efficiency and accuracy up to now.

On the other hand, in order to avoid the identification of failure mode and the computation of failure probability from mode probability, the Monte Carlo simulation can be directly used to deal with the calculation of system reliability (Grim-

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melt and Schueller 1982; Melchers 1994), but the computation becomes too expensive to be applied to large structural systems or those with low failure probability. Furthermore, direct application of the importance sampling is not feasible since the limit state functions and the design point are not known (Melchers 1987).

Therefore, to develop an efficient procedure of system reliability evaluation for ductile frame structures, the obstacle in failure mode identification and the failure probability computation should be avoided.

ESTABLISHMENT OF PERFORMANCE FUNCTION

As described previously, the definition of performance function in (1) has the following weaknesses:

1. All or part of the failure modes and their performance functions are needed, and it is not always easy to identify them.
2. It is difficult to compute the failure probability even when the failure modes are known, because of the difficulty in obtaining the sensitivity of performance function and the correlation among the failure modes.

To avoid these weaknesses, one may readily associate to define the performance function as shown in (2).

$$G = d_c - d(\mathbf{M}, \mathbf{P}) \tag{2}$$

where d_c = threshold value; and $d(\mathbf{M}, \mathbf{P})$ = deformation of the ductile frame. The definition of threshold value d_c and the computation of ductile deformation $d(\mathbf{M}, \mathbf{P})$ will be needed, the simplicity of limit analysis will be lost, and also, it does not include the characteristics of the system reliability.

To define a performance function without these weaknesses on the base of the assumptions described in the previous section, one can firstly observe the limit state function under only one load P. Because the structure will become a kinematically admissible mechanism when the load P increases to the utmost load, the performance function can be described as

$$G(\mathbf{M}, \mathbf{P}) = u_P(\mathbf{M}) - P \tag{3}$$

where $u_P(\mathbf{M})$ = utmost load. The value of $u_P(\mathbf{M})$ is dependent on $\mathbf{M}$, and a different failure mode may happen with different $\mathbf{M}$.

In limit analysis, the limit load is generally described as the times of initial load and a load factor; therefore, (3) can be described as follows:

$$G(\mathbf{M}, P) = \lambda(\mathbf{M}, P)P - P \tag{4}$$

where $\lambda(\mathbf{M}, P)$ = load factor; and P = initial load.

In (4), the amount of P does not influence the shape of the limit state surface $G(\mathbf{M}, P) = 0$; therefore, the same reliability analysis results will be obtained if it is written as follows:

$$G(\mathbf{M}, \mathbf{P}) = \lambda(\mathbf{M}, P) - 1 \tag{5}$$

For multiple load, the limit state function is defined in the load space, and it cannot be dealt with as (4). One can divide the load space into various load paths and consider one load path. For example, for a frame structure with two load $\mathbf{P} = [P_1, P_2]$, considering a load path $\mathbf{P}(\theta_i)$ where θ_i is defined as $tg\theta_i = P_2/P_1$, the utmost load will be reached when the load increased along this load path. The performance function can be written as follows, as already described:

$$G_i(\mathbf{M}, \mathbf{P}) = \lambda(\mathbf{M}, \mathbf{P})\mathbf{P}(\theta_i) - \mathbf{P}(\theta_i) \tag{6}$$

where $\lambda(\mathbf{M}, \mathbf{P})$ = load factor; and $\mathbf{P}(\theta_i)$ = load path.

In (6), the amount of $\mathbf{P}(\theta_i)$ does not influence the shape of the limit state surface $G(\mathbf{M}, \mathbf{P}) = 0$; therefore, the same reliability analysis results will be obtained if it is written as follows:

$$G(\mathbf{M}, \mathbf{P}) = \lambda(\mathbf{M}, \mathbf{P}) - 1 \tag{7}$$

Different failure modes will happen with different $\mathbf{M}$ and different load path $\mathbf{P}(\theta_i)$, but (7) always holds true with any load path. With given $(\mathbf{M}, \mathbf{P})$, only one value of $\lambda(\mathbf{M}, \mathbf{P})$ can be obtained, and (7) is the general form of performance function for ductile frame structures.

Because utmost load is obtained as the minimum load of formulating a kinematically admissible mechanism, the values of (1) and (6) are the same, i.e., limit state surface expressed by (1) and (7) are the same, although their performance functions are different. The limit state surface expressed by (7) is the inner connotative surface of the limit state surfaces corresponding to all the failure modes.

Limit Analysis

To obtain the load factor, limit analysis is conducted using compact procedure (Aoyama 1988). In this procedure, the equilibrium equation is taken to be the object function, and the utmost strength is taken as the limit condition. The mechanisms can be identified from the structural analysis when the total stiffness matrix become singular. The limit analysis is defined as a problem of obtaining the maximum load factor that satisfies the equilibrium equation and the limit condition using the linear programming method. The equilibrium equation is described as

$$\lambda\mathbf{P} = \mathbf{HR} \tag{8}$$

where $\mathbf{P}$ = load vector; λ = load factor; $\mathbf{R}$ = vector of member moment; and the utmost value of $\mathbf{R}$ is the vector of moment capacities $\mathbf{M}$. $\mathbf{H}$ is the coefficient matrix of the equilibrium equation.

The Gauss-Jordan method (Livesley 1976) is applied to solve (8), and the following two steps are repeated until the load factor λ reaches its maximum. Then $\lambda(\mathbf{M}, \mathbf{P})$ in (7) will be obtained.

In step 1, divide $\mathbf{R}$ into fundamental variables and nonfundamental variables according to the contents of $\mathbf{H}$. Change the fundamental variables. Increase the load factor until the utmost value (moment capacity) of a fundamental variable is reached.

In step 2, in order to increase the load factor further, exchange the fundamental variables and the nonfundamental variables.

APPLICATION OF RESPONSE SURFACE APPROACH

Although the form of (7) is very simple, it includes a complicated function $\lambda(\mathbf{M}, \mathbf{P})$. The function has no explicit form, and it is difficult to obtain its sensitivity. To avoid these problems, the response surface approach will be applied.

Response Surface Approach

The response surface approach is a collection of statistical analysis methods that examine the relationship between experimental response and variations in the values of input variables. Developed by research scientists performing experiments in biology and agriculture (Box et al. 1978; Draper and Smith 1981), it has been applied to create and analyze the statistical model of performance function in structural reliability that is difficult for many researchers to study directly.

In reliability analysis of practical structures, the performance function $G(\mathbf{X})$ is generally described only by using implicit forms; here, $\mathbf{X} = \{\mathbf{M}, \mathbf{P}\}$ expresses the vector of random variables. In this case, it is time-consuming to evaluate the probability of the limit state being exceeded. The basic concept of the so-called response surface approach is to replace the original implicit performance function with an approximated explicit function in terms of basic random variables. With an eye toward the accuracy as well as the high cost of repeated computation of $G(\mathbf{X})$, second-degree polynomials are generally used (Wong 1984, 1985; Bucher et al. 1988; Bucher and Bourgund 1990; Faravelli 1989). For simplification, the following equation is used in this paper:

$$G' = a + \sum_{i=1}^{n} b_i x_i + \sum_{i=1}^{n} c_i x_i^2 \tag{9}$$

where a, b_i, c_i = progressive coefficients with number of $2n + 1$; and n = number of random variables.

To improve the accuracy of the response surface approach, Bucher and Bourgund (1990) suggested an alternative process of selecting the fitting points. In the first step of this algorithm, the mean vector is selected as the center point, then the response surface obtained is used to estimate the design point. In the next step, the new center point is chosen on a straight line from the initial center point to the design point obtained so that $G(\mathbf{X}) = 0$ at the new center point from liner interpolation, i.e.

$$\mathbf{X}_M = \mu + (\mathbf{X}_D - \mu)\frac{G(\mu)}{G(\mu) - G(\mathbf{X}_D)} \tag{10}$$

where μ = mean vector; and $\mathbf{X}_D$, $\mathbf{X}_M$ = design point and new center point, respectively.

This process is assumed to guarantee that the fitting points chosen from the new center point include the information of the original failure surface sufficiently. Some improvements have been proposed (Rajashekhar and Ellingwood 1993; Liu and Moses 1994; Yao and Wen 1996).

The suggested fitting points for obtaining $G'(\mathbf{X})$ lie along the x_i-axis. The points chosen are mean values of the random variables and $x_i = \mu_i \pm d_i\sigma_i$, in which d_i is an arbitrary factor and σ_i is the standard deviation of x_i. The progressive coefficients are determined using $(2n + 1)$ values of $G(\mathbf{X})$ at these points.

Computation of Failure Probability

After the approximated performance function is determined, the FORM can be easily applied to obtain the Hosefer-Lind reliability index, because the sensitivity coefficient can be obtained directly from (9). Because (9) is an approximation of the inner connotative surface of the limit state surfaces corresponding to all the failure modes, the HL reliability index of (9) is equivalent to that of the most likely failure mode (the failure mode that has the minimum Hosefer-Lind reliability index). Therefore, to capture the effect of other failure modes and evaluate system reliability of frame structures properly, reliability corresponding to (9) should be evaluated relatively accurately. The second-order reliability method or importance sampling method is not difficult to apply to obtain the failure probability, because the design point has been obtained by FORM. For convenience, the following empirical reliability index (Zhao and Ono, to be published, 1997) is used in this paper:

$$\beta_S = \begin{cases} \left[1 - \dfrac{K_s}{3\beta_F + 3(n-1)/K_s + 1}\right]\beta_F + \dfrac{1}{2}K_s & K_s \ge 0 \\ \left[1 - \dfrac{K_s^2}{3(n - \beta_F + 3)}\right]\beta_F + \dfrac{1}{2}K_s & K_s < 0 \end{cases} \tag{11}$$

where K_s = sum of the principle curvatures of the limit state surface described as

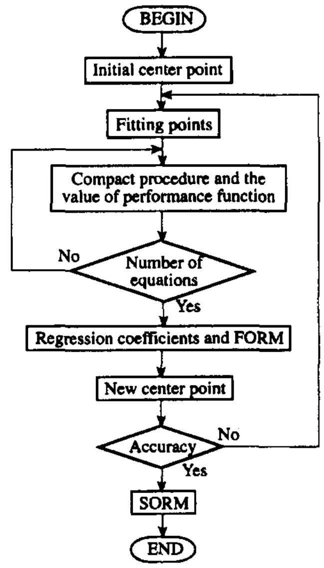

FIG. 1. Flowchart of Procedure

$$K_s = \sum_{i=1}^{n} b_{ii} - \alpha^T \mathbf{B}\alpha \tag{12}$$

$$\alpha = -\frac{\nabla G(\mathbf{U}^*)}{|\nabla G(\mathbf{U}^*)|}; \qquad [\mathbf{B}] = \frac{\nabla^2 G(\mathbf{U}^*)}{|\nabla G(\mathbf{U}^*)|}$$

where $\mathbf{U}^*$ = design point in u-space; ∇G = gradients of G; $\nabla^2 G$ = second derivatives of G; n = number of random variables; β_F = first-order reliability index; β_S = second-order reliability index; α = directional vector at design point in u-space; $\mathbf{B}$ = Hessian matrix at design point in u-space; and b_{ii} = diagonal element of $\mathbf{B}$.

The flowchart of the proposed procedure is illustrated in Fig. 1, which includes two steps. The first step is evaluating the maximum load factor using compact procedure and determine the progressive coefficients in (9). The second step is computing the reliability index by FORM/SORM.

Because of the introduction of performance function (7), it also becomes easy to conduct reliability analysis of ductile frame considering geometric second-order effects, if one replaces the limit analysis by second-order limit analysis.

Needless to say, because the performance function of (7) is approximated by a second-degree polynomial, the procedure is not applicable if the limit state surface has the shape of a sawtooth. But generally, there is a large number of potential failure modes; the fear of limit state surface of sawtooth shape is almost not necessary. As described in the next section, the proposed procedure generally gives accurate enough results whether there is a dominant failure mode or not.

COMPUTATIONAL EXAMPLE AND INVESTIGATION

First Example

Because the performance function of ductile frame is generally defined as the minimum value of performance functions corresponding to the failure modes, and in order to investigate the efficiency of the proposed procedure, the performance function in the first example is defined as the minimum value of eight linear performance functions listed in Table 1, and the limit state surface is shown in Fig. 2. Assume x_1, x_2 are independent standard normal random variables.

The performance function is approximated using the second-degree polynomial defined in (7). The center point is taken to be the mean value of the random variables, and the other fitting points are taken to be the points that have distance σ from the center point.

The convergency is reached in four iterations with tolerance criteria of $|(\beta_i - \beta_{i-1})/\beta_i| < 10^{-3}$, in which β_i is the first-order reliability index obtained in ith iteration. The performance functions obtained during the four iterations are listed in Table 2, and the corresponding response surfaces are shown in Fig. 2. From these results, one can see:

1. The reliability index obtained in the first iteration, 2.991, is not sufficient as the approximation of the minimum reliability index listed in Table 1. The reliability index obtained in the fourth iteration, 2.831, is quite near to 2.828, which is the minimum value of the reliability index listed in Table 1. Therefore, to obtain the approximate value of the reliability index, it is necessary to conduct the iteration up to convergency.
2. From Fig. 2, one can see that the obtained response surface is a good approximation for the inner connotative line of the eight limit state lines.
3. The design point of the response surface obtained in the fourth iteration is [−2.09, 1.91]; it is matching the design point [−2.0, 2.0] corresponding to the minimum reliability index listed in Table 1.

TABLE 1. Performance Function of First Example

Performance function (1)	Design point (2)	β (3)
$G_1 = 1.4x_1 - 2x_2 + 7.2$	−1.691, 2.416	2.949
$G_2 = 2x_1 - 2x_2 + 8$	−2.000, 2.000	2.828
$G_3 = 2.6x_1 - 2x_2 + 9.3$	−2.247, 1.729	2.835
$G_4 = -1.5x_1 - 2x_2 + 10$	2.400, 3.200	4.000
$G_5 = 4x_1 - 2x_2 + 14$	−2.800, 1.400	3.130
$G_6 = 0.7x_1 - 2x_2 + 6.8$	−1.060, 3.029	3.209
$G_7 = -0.5x_1 - 2x_2 + 8$	0.941 3.765	3.881
$G_8 = -2x_1 - 2x_2 + 11$	2.750, 2.750	3.889

Note: $G = \min\{G_1, G_2, G_3, G_4, G_5, G_6, G_7, G_8\}$.

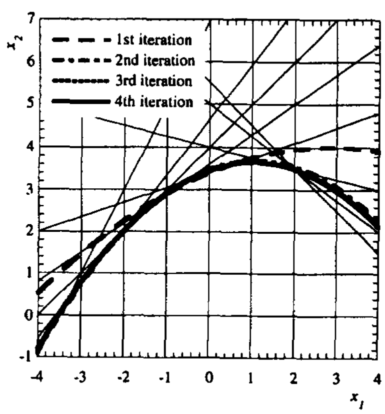

FIG. 2. Response Surfaces in First Example

TABLE 2. Response Surface Function and Its FORM Computational Results

Iteration number (1)	Performance function (2)	β (3)	P_f (4)	Design point (5)
1	$G'_{1st} = 6.8 + 0.85x_1 - 2x_2 - 0.15x_1^2$	2.991	1.391×10^{-3}	−1.668, 2.483
2	$G'_{2nd} = 6.944 + 0.783x_1 - 2x_2 - 0.35x_1^2 - 2.384 \times 10^{-7}x_2^2$	2.817	2.422×10^{-3}	−2.103, 1.875
3	$G'_{3rd} = 6.897 + 0.74x_1 - 2x_2 - 0.35x_1^2 - 1.192 \times 10^{-7}x_2^2$	2.832	2.311×10^{-3}	−2.091, 1.911
4	$G'_{4th} = 6.898 + 0.741x_1 - 2x_2 - 0.35x_1^2$	2.831	2.317×10^{-3}	−2.090, 1.910

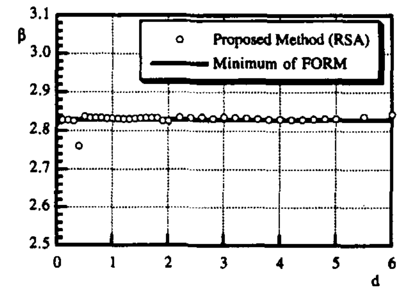

FIG. 3. Reliability Index with Different Fitting Points

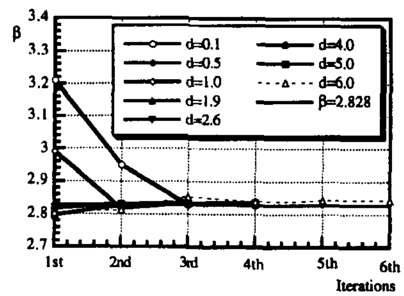

FIG. 4. Convergency with Different Fitting Points

Monte Carlo simulation for the performance function listed in Table 1 with 100,000 samples is conducted. The failure probability P_F is obtained as 2.93×10^{-3} with its standard deviation of 1.71×10^{-4}. The corresponding reliability index of the Monte Carlo simulation is obtained as 2.76. Using the obtained final performance function listed in Table 2, the SORM result using (11) can be given as $\beta_S = 2.773$. One can see the SORM reliability index is close to that of Monte Carlo simulation.

In order to investigate the affection of fitting points, reliability indices obtained with different fitting points are shown in Fig. 3, where d is the distance from the center point described as the times of standard deviation. From Fig. 3, one can see that the reliability indices obtained with different fitting points (different d in the figure) almost coincide with the minimum value obtained by FORM, that is to say, the reliability index obtained by the proposed method is almost not affected by the fitting points. To investigate the convergency of the procedure, the stage reliability indices obtained in each iteration are shown in Fig. 4, from which one can see that although the reliability indices in the first iteration are much different with different fitting points, the final reliability indices are kept almost the same with the different fitting points.

Second Example

The second example is a frame structure of one story and one bay, shown in Fig. 5 with the probabilistic member strength and load listed in Table 3. For comparison, the failure mode and the corresponding performance functions obtained from stochastic limit analysis (Ono et al. 1997) are listed in (13):

$$G_1 = M_1 + 3M_2 + 2M_3 - 15S_1 - 10S_2; \quad (\beta = 3.551) \tag{13a}$$

$$G_2 = 2M_1 + 2M_2 - 15S_1; \quad (\beta_{min} = 3.247) \tag{13b}$$

$$G_3 = M_1 + M_2 + 4M_3 - 15S_1 - 10S_2; \quad (\beta = 3.848) \tag{13c}$$

$$G_4 = 2M_1 + M_2 + M_3 - 15S_1; \quad (\beta = 3.562) \tag{13d}$$

$$G_5 = M_1 + M_2 + 2M_3 - 15S_1; \quad (\beta = 3.784) \tag{13e}$$

$$G_6 = M_1 + 2M_2 + M_3 - 15S_1; \quad (\beta = 3.562) \tag{13f}$$

where all the probabilistic member strength and load are as-

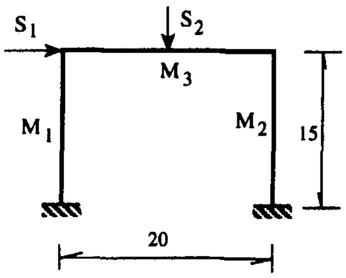

FIG. 5. One-Story One-Bay Frame

TABLE 3. Variables in Second Example

Variables (1)	Mean (2)	Coefficient of variation (3)
M_1, M_2	500	0.15
M_3	667	0.15
S_1	50	0.30
S_2	100	0.10

sumed to be lognormal random variables. The values in the parentheses are the FORM reliability index corresponding to each failure mode. The most likely collapse mode is G_2 with the minimum FORM reliability index of 3.247, and it is dominant to other modes.

The reliability evaluation is conducted using the proposed method shown in Fig. 1. The performance functions listed in (13) are only for comparison; they need not be used in the procedure. The performance function of (7) is used and $\lambda(\mathbf{M}, \mathbf{P})$ is obtained by limit analysis.

To investigate the convergency of the procedure, the computations are conducted with different fitting points. The number of iterations used in each computation, the obtained response surface function, and the corresponding FORM results are listed in Table 4, where $d_M\sigma$ is the distance from fitting points in M axes to center point, and $d_S\sigma$ is that in S axes to center point. Different $d_M\sigma$ and $d_S\sigma$ express different fitting points.

From Table 4, one can see that the computations reach convergency in few iterations in all cases of different fitting points. Although there are some slight differences among the response surfaces obtained with different fitting points, all of the response surfaces are similar to the performance function corresponding to the most likely collapse mode G_2 in the following characteristics:

1. They only contain the basic variable of M_1, M_2, S_1.
2. The coefficients of M_1 and M_2 are the same.
3. All the FORM reliability indices corresponding to all the response surfaces are almost the same. They are equal to or nearly equal to 3.247, which is the same as the minimum value listed in (13).

Monte Carlo simulation for limit analysis with 100,000 samples is conducted. The failure probability P_F and the corresponding reliability index are listed in Table 4, from which one can see that the reliability index 3.247 obtained from the response surface approach is almost the same as the result of 3.25 obtained by the Monte Carlo simulation, because in this example, the most likely failure mode G_2 is dominant to other failure modes.

Third Example

The third example is a frame structure with two stories and two bays as shown in Fig. 6, with the probabilistic member strength and load listed in Table 5. For comparison, the failure modes and the corresponding performance functions are listed in (14):

$$G_1 = 2M_1 + 2M_2 + 2M_3 - 15S_1 - 15S_2; \quad (\beta_{\min} = 3.100) \quad (14a)$$

$$G_2 = M_6 + M_7 + 2M_8 - 10S_5; \quad (\beta = 3.205) \quad (14b)$$

$$G_3 = M_3 + 3M_5 - 10S_4; \quad (\beta = 3.291) \quad (14c)$$

$$G_4 = M_7 + 3M_8 - 10S_5; \quad (\beta = 3.304) \quad (14d)$$

$$G_5 = 2M_1 + 2M_2 + M_3 + M_5 - 15S_1 - 15S_2; \quad (\beta = 3.681) \quad (14e)$$

$$G_6 = M_6 + 3M_7 + 2M_8 - 15S_1; \quad (\beta = 3.725) \quad (14f)$$

where the figures in the parentheses are the FORM reliability indices corresponding to each failure mode. The most likely collapse model is G_1 with the minimum FORM reliability index of 3.100. There is no significant dominant collapse mode in this example.

Using the proposed procedure, the computation reaches the convergency in few iterations, and the obtained response surfaces with different fitting points are listed in Table 6, with FORM/SORM reliability index [obtained using (11)] corresponding to each response surface.

From Table 6, one can see that despite the case of $d_M = d_S = 1.0$, in general cases, the response surfaces obtained with different fitting points are all similar to the performance function corresponding to the most likely collapse mode G_1 in the three characteristics described in Example 2. The FORM reliability indices corresponding to the response surfaces are approximately equal to 3.100, which is the minimum FORM reliability index listed in (14).

Monte Carlo simulation for limit analysis with 100,000 samples is conducted. The failure probability P_F and the corre-

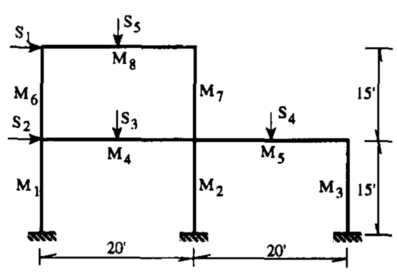

FIG. 6. Two-Story Two-Bay Frame in Third Example

TABLE 4. Response Surfaces Obtained with Different Fitting Points in Second Example

Fitting Point		Response Surface Approach			
$d_M\sigma$ (1)	$d_S\sigma$ (2)	Response surface (3)	β_F (4)	P_F (5)	Iterations (6)
1.0σ	1.0σ	$10^4G' = 10{,}330 + 11.3M_1 + 11.3M_2 - 259S_1 + 0.734S_1^2$	3.247	5.837×10^{-4}	6
1.0σ	0.5σ	$10^4G' = 10{,}460 + 11.3M_1 + 11.3M_2 - 259S_1 + 0.733S_1^2$	3.247	5.828×10^{-4}	6
0.5σ	1.0σ	$10^4G' = 11{,}978 + 11.9M_1 + 11.9M_2 - 293S_1 + 0.870S_1^2$	3.240	5.980×10^{-4}	7
1.6σ	1.6σ	$10^4G' = 7{,}060 + 11.4M_1 + 11.4M_2 - 205S_1 + 0.508S_1^2$	3.247	5.837×10^{-4}	8

Note: Monte Carlo simulation of limit analysis: $\beta = 3.25$; $P_F = 5.7 \times 10^{-4}$; $\sigma_{P_F} = 7.5 \times 10^{-5}$.

sponding reliability index are listed in Table 6, from which one can see that almost all the reliability indices obtained from the response surface approach are close to the result of 2.82 obtained by the Monte Carlo simulation, i.e., the response surface can be accepted as the approximated limit state surface of the frame structure and the reliability index can be accepted as that of the structural system.

In the case of $d_M = d_S = 1.0$, the obtained response surface is similar to the performance function of G_3 in the following characteristics:

1. It only contains the basic variable of M_3, M_5, S_4.
2. The coefficient of M_5 is three times of that of M_3.
3. The reliability indices are nearly the same.

That is to say, the response surface is locally converged into the limit state surface G_3. In fact, the possibility of local convergency is a common weakness of an iteration method such as FORM. This is the precaution one needs to take when using this method. To avoid this weakness, one should check the obtained design point using different fitting points.

Fourth Example

The fourth example is a practical frame structure with six stories and three bays as shown in Fig. 7, with the probabilistic member strength and load listed in Table 7. The reliability evaluation is conducted using the proposed method with fitting points of $d_M = d_S = 1.0\sigma$. The computation reaches convergency in four iterations, and the response surface is obtained as

$$10^4G' = 6{,}384 + 16.3M_1 + 16.3M_4 + 16.3M_7 + 16.3M_{10} + 16.3M_{13} + 8.16M_{14} + 8.16M_{15} - 32.9S_2 - 69.8S_3 - 115.2S_4 - 180.8S_5 - 407.6S_6 + 0.099S_2^2 + 0.397S_3^2 + 0.895S_4^2 + 1.604S_5^2 + 2.766S_6^2 \quad (15)$$

From (15), one can see that the response surface function only contains the basic variables of M_1, M_4, M_7, M_{10}, M_{13}, M_{14}, M_{15}, S_2, S_3, S_4, S_5, S_6. The coefficients of M_1, M_4, M_7, M_{10}, M_{13} are the same, and those of M_{14}, M_{15} are the same. From these variables included in the performance function (15), one can understand the sections where plastic hinges formed, and can obtain the most likely failure mode as shown in Fig. 8. Using the equation of virtual work, the performance function corresponding to the failure mode is obtained as (16). The same results are also obtained using stochastic limit analysis.

$$G_{\min} = 2M_1 + 2M_4 + 2M_7 + 2M_{10} + 2M_{13} + M_{14} + M_{15} - 3.8S_2 - 7.6S_3 - 11.4S_4 - 15.2S_5 - 19S_6 \quad (16)$$

All the characteristics of (15) described previously can be also found in (16). The FORM reliability index corresponding to (15) is equal to 2.640, while that corresponding to (16) is equal to 2.651, i.e., (15) and (16) have similar FORM reliability indices. That is to say, the procedure can be also used to obtain the most likely failure mode of frame structures.

The influences by fitting points are also checked with the results listed in Table 8, from which one can see that the computational results are almost not influenced by fitting points.

In order to check the results of response surface approach, a Monte Carlo simulation for limit analysis with 10,000 samples is conducted. The failure probability P_F and the corresponding reliability index are listed in Table 8, from which one can see that all the reliability indices obtained from the

TABLE 5. Random Variables in Third Example

Variables (1)	Distribution (2)	Mean (3)	Coefficient of variation
M_1, $-M_3$	Lognormal	7	0.15
M_6, M_7	Lognormal	70	0.15
M_4	Lognormal	150	0.15
M_5	Lognormal	120	0.15
M_8	Lognormal	90	0.15
S_1	Lognormal	5	0.25
S_2	Lognormal	10	0.25
S_3	Lognormal	26.5	0.15
S_4	Lognormal	18	0.25
S_5	Lognormal	14	0.25

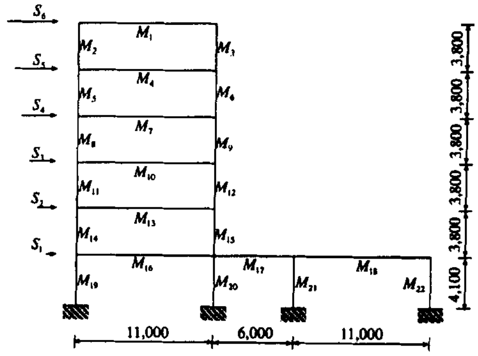

FIG. 7. Six-Story Three-Bay Frame in Fourth Example

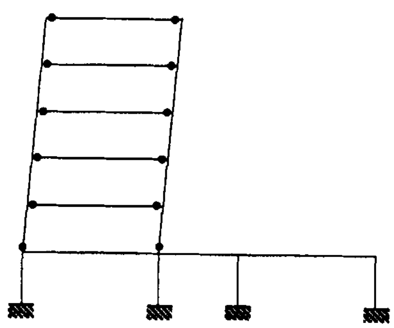
FIG. 8. Most Likely Failure Mode in Fourth Example

TABLE 6. Response Surfaces Obtained with Different Fitting Points in Third Example

Fitting Point		Response Surface Approach			
$d_M\sigma$ (1)	$d_S\sigma$ (2)	Response surface (3)	FORM (4)	SORM (5)	Iterations (6)
1.0σ	1.0σ	$10^4G' = 11{,}336 + 30.7M_3 + 92.2M_5 - 983.4S_4 + 10.1S_4^2$	$\beta = 3.224$; $P_F = 6.324 \times 10^{-4}$	$\beta = 2.839$; $P_F = 2.261 \times 10^{-4}$	4
0.8σ	1.3σ	$10^4G' = 7{,}047 + 54.4M_1 + 54.4M_2 + 54.4M_3 - 620.2S_1 - 1{,}082S_2 + 16.8S_1^2 + 17.7S_2^2$	$\beta = 3.095$; $P_F = 9.831 \times 10^{-4}$	$\beta = 2.935$; $P_F = 1.666 \times 10^{-3}$	4
1.0σ	1.5σ	$10^4G' = 7{,}419 + 54.1M_1 + 54.1M_2 + 54.1M_3 - 613.5S_1 - 1{,}103S_2 + 16.6S_1^2 + 17.9S_2^2$	$\beta = 3.094$; $P_F = 9.878 \times 10^{-4}$	$\beta = 2.936$; $P_F = 1.659 \times 10^{-3}$	7
1.5σ	1.5σ	$10^4G' = 7{,}419 + 54.1M_1 + 54.1M_2 + 54.1M_3 - 613.5S_1 - 1{,}103S_2 + 16.6S_1^2 + 17.9S_2^2$	$\beta = 3.094$; $P_F = 9.878 \times 10^{-4}$	$\beta = 2.938$; $P_F = 1.648 \times 10^{-3}$	8

Note: Monte Carlo simulation of limit analysis: $\beta = 2.82$; $P_F = 2.41 \times 10^{-3}$; $\sigma_{P_F} = 1.6 \times 10^{-4}$.

TABLE 7. Random Variables in Fourth Example

Variables (1)	Distribution (2)	Mean (3)	Coefficient of variation (4)
$M_1, M_4, M_7, M_{17}, M_{18}$	Lognormal	$90.8t - m$	0.1
M_2, M_3, M_5, M_6	Lognormal	$145.2t - m$	0.1
M_8, M_9, M_{21}, M_{22}	Lognormal	$145.2t - m$	0.1
M_{10}, M_{13}, M_{16}	Lognormal	$103.4t - m$	0.1
M_{11}, M_{12}, M_{14}	Lognormal	$162.8t - m$	0.1
M_{15}, M_{19}, M_{20}	Lognormal	$162.8t - m$	0.1
S_1	Lognormal	$2.5t$	0.5
S_2	Lognormal	$5.0t$	0.5
S_3	Lognormal	$7.5t$	0.5
S_4	Lognormal	$10.0t$	0.5
S_5	Lognormal	$12.5t$	0.5
S_6	Lognormal	$15.0t$	0.5

TABLE 8. Computational Results for Fourth Example

Fitting Point		Response Surface Approach	
$d_M\sigma$ (1)	$d_S\sigma$ (2)	FORM (3)	SORM (4)
1.0σ	1.0σ	$\beta = 2.640$; $P_F = 4.14 \times 10^{-3}$	$\beta = 2.514$; $P_F = 5.962 \times 10^{-3}$
1.5σ	0.3σ	$\beta = 2.651$; $P_F = 4.02 \times 10^{-3}$	$\beta = 2.586$; $P_F = 4.858 \times 10^{-3}$
1.2σ	0.5σ	$\beta = 2.650$; $P_F = 4.02 \times 10^{-3}$	$\beta = 2.581$; $P_F = 4.931 \times 10^{-3}$
0.8σ	0.8σ	$\beta = 2.647$; $P_F = 4.02 \times 10^{-3}$	$\beta = 2.548$; $P_F = 5.423 \times 10^{-3}$

Note: Monte Carlo simulation of limit analysis: $\beta = 2.550$; $P_F = 5.3 \times 10^{-3}$; $\sigma_{P_F} = 7.3 \times 10^{-4}$.

response surface approach are close to the result of 2.55 obtained by the Monte Carlo simulation, i.e., the response surface can be accepted as the approximated limit state surface of the framed structure and the reliability index can be accepted as that of the structural system.

CONCLUSIONS

A performance function that is independent of failure mode and load path is defined and a system reliability evaluation procedure is proposed. It is found that:

1. To obtain the approximate value of the reliability index, it is necessary to conduct the iteration to convergency.
2. The response surface approach can give a good approximation of the inner connotative surface of the limit state surfaces. The proposed procedure can be also used to obtain the most likely failure mode of frame structures.
3. The FORM reliability index corresponding to the response surface is a good approximation of the FORM reliability index corresponding to the most likely collapse mode. To heighten the evaluation accuracy of system reliability, SORM needs to be used. Basically, SORM is capturing the effects of failure modes other then the most likely failure mode.
4. Although there are some slight differences among the response surfaces obtained with different fitting points, reliability evaluation results are almost the same, i.e., the reliability results are generally not significantly influenced by fitting points.
5. The proposed procedure has good efficiency and enough accuracy. The difficulty in both the failure mode identification and the failure probability computation can be avoided.
6. There is a slight fear of local convergency in the proposed procedure. To ensure its accuracy and reliability, one needs to check the results using different fitting points.

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APPENDIX I. REFERENCES

Ang, A. H.-S., and Ma, H. F. (1981). "On the reliability of structural systems." *Proc., 3rd Int. Conf. on Struct. Safety and Reliability*, Elsevier, Amsterdam, The Netherlands, 295–314.
Aoyama, H. (1988). *Structural analysis by method of matrix*. Paifukan Press, Tokyo, Japan (in Japanese).
Bennett, R. M., and Ang, A. H.-S. (1987). "Formulation of structural system reliability." *J. Engrg. Mech.*, ASCE, 112(11), 1135–1151.
Box, G. E. P., Hunter, W. G., and Hunter, J. S. (1978). *Statistical for experimenters*. John Wiley & Sons, Inc., New York, N.Y.
Bucher, C. G., and Bourgund, U. (1990). "A fast and efficient response surface approach for structural reliability problems." *Struct. Safety*, 7, 57–66.
Bucher, C. G., Chen, Y. M., and Schueller, G. I. (1988). "Time variant reliability analysis utilizing response surfaces approach." *Reliability and optimization of structural system '88*, P. Thoft-Christensen, ed., Springer-Verlag, Berlin, Germany, 1–14.
Cornell, C. A. (1966). "Bounds on the reliability of structural systems." *J. Struct. Div.*, ASCE, 93(1), 171–200.
Draper, N., and Smith, H. (1981). *Applied regression analysis*, 2nd Ed., John Wiley & Sons, Inc., New York, N.Y.
Faravelli, L. (1989). "Response surface approach for reliability analysis." *J. Engrg. Mech.*, ASCE, 115(12), 2763–2781.
Grimmelt, M. J., and Schueller, G. I. (1982). "Benchmark study on methods to determine collapse failure probabilities of redundant structures." *Struct. Safety*, 1, 93–106.
Idota, H., Ono, T., and Hayakawa, Y. (1991). "A study on analysis of highly redundant structural systems." *Proc., 2nd Japanese Conf. on Struct. Safety and Reliability*. Japan Society of Civil Engineers, Tokyo, Japan, 497–502 (in Japanese).
Liu, Y. W., and Moses, F. (1994). "A sequential response surface method and its application in the reliability analysis of aircraft structural systems." *Struct. Safety*, 16, 39–46.
Livesley, R. K. (1976). *Matrix methods of structural analysis*, 2nd Ed., Pergamon Press, New York, N.Y.
Melchers, R. E. (1987). *Structural reliability—analysis and prediction*. John Wiley & Sons, Inc., New York, N.Y.
Melchers, R. E. (1994). "Structural system reliability assessment using directional simulation." *Struct. Safety*, 16, 23–37.
Melchers, R. E., and Tang, L. K. (1985). "Failure mode in complex stochastic systems." *Proc., 4th Int. Conf. on Struct. Safety and Reliability*, Vol. 1, IASSAR, New York, N.Y., 97–106.
Moses, F. (1982). "System reliability developments in structural engineering." *Struct. Safety*, 1, 3–13.
Murotsu, Y., Okada, H., Grimmelt, M. J., Yonezawa, M., and Taguchi, K. (1984). "Automatic generation of stochastically dominant failure modes of frame structures." *Struct. Safety*, 2, 17–25.
Nafday, A. M. (1987). "Failure mode identification for structural frames." *J. Struct. Engrg.*, ASCE, 113(7), 1415–1432.
Ohi, K. (1991). "Stochastic limit analysis of framed structures." *Proc., 2nd Japanese Conf. on Struct. Safety and Reliability*, Japan Society of Civil Engineers, Tokyo, Japan, 675–678 (in Japanese).
Ono, T., Idota, H., and Dozuka, A. (1990). "Reliability evaluation of structural systems using higher-order moment standardization technique." *J. Struct. Constr. Engrg.*, Tokyo, Japan, No. 418, 71–79 (in Japanese).
Ono, T., Zhao, Y. G., and Yoshihara, K. (1997). "Probabilistic evaluation of COF of frame structures using stochastic limit analysis." *J. Struct. Engrg.*, Tokyo, Japan, 43B, 301–308 (in Japanese).
Rajashekhar, M. R., and Ellingwood, B. R. (1993). "A new look at the response surface approach for reliability analysis." *Struct. Safety*, 12, 205–220.
Schueller, G. I., and Stix, R. (1987). "A critical appraisal of methods to determine failure probabilities." *Struct. Safety*, 4, 293–309.
Thoft-Christensen, P., and Murotsu, Y. (1986). *Application of structural systems reliability theory*. Springer, Berlin, Germany.
Wong, F. S. (1984). "Uncertainties in dynamic soil-structure interaction." *J. Engrg. Mech.*, ASCE, 110(2), 308–324.
Wong, F. S. (1985). "Slope reliability and response surface method." *J. Geotech. Engrg.*, ASCE, 111(1), 32–53.

Xiao, Q., and Mahadeven, S. (1994). "Fast faiure mode identification for ductile structural system reliability." *Struct. Safety*, 13, 207–226.
Yao, H. J., and Wen, Y. K. (1996). "Response surface method for time-variant reliability analysis." *J. Struct. Engrg.*, ASCE, 122(2), 193–201.
Zhao, Y. G., and Ono, T. (1997). "An empirical reliability index based on SORM." *Proc. 7th Int. Conf. on Struct. Safety and Reliability*, Kyoto, Japan, to be published.
Zimmerman, J. J., Ellis, J. H., and Corotis, R. B. (1993). "Stochastic optimization models for structural reliability analysis." *J. Struct. Engrg.*, ASCE, 119(1), 223–239.

APPENDIX II. NOTATION

The following symbols are used in this paper:

$\mathbf{B}$ = Hessian matrix at design point in u-space;
b_{ii} = diagonal element of $\mathbf{B}$;
d_c = threshold value;
$d(\mathbf{M}, \mathbf{P})$ = deformation of the ductile frame;
$d_M\sigma$ = distance from fitting points in M axes to center point;
$d_S\sigma$ = distance from fitting points in S axes to center point;
$G_i(\mathbf{M}, \mathbf{P})$ = performance function corresponding to ith failure mode;
$\mathbf{H}$ = coefficient matrix of the equilibrium equation;
K_s = sum of the principle curvatures of the limit state surface;
$\mathbf{M}$ = vector of moment capacities of the structure;
n = number of random variables;
$\mathbf{P}$ = load vector;
P_F = failure probability;
$\mathbf{R}$ = vector of member moment;
$\mathbf{U}^*$ = design point in u-space;
$\mathbf{X}$ = $\{\mathbf{M}, \mathbf{P}\}$ expresses the vector of random variables;
α = directional vector at design point in u-space;
β = reliability index;
β_F = first-order reliability index;
β_S = second-order reliability index;
$\lambda(\mathbf{M}, \mathbf{P})$ = load factor;
σ = standard deviation;
σ_{P_F} = standard deviation of failure probability;
∇G = gradients of G; and
$\nabla^2 G$ = second derivatives of G.